Abstract—We consider a sensor communication scenario where a sensor node communicates information to a data sink. To aid the sensor, the data sink transmits channel state information (CSI). To obtain the CSI, however, a fixed receive processing energy is incurred by the sensor node. Given a choice, the sensor node may choose not to obtain the CSI if the receive processing energy does not lead to a larger reduction in the transmit energy. In this paper, we study the tradeoff involved in this choice. Specifically, we design the optimal transmit strategy so that the total expected transmit and receive power of the sensor are minimized subject to a target outage probability. We focus on the case in which the feedback packet conveys one bit of CSI to the sensor nodes. Closed-form expressions of suboptimal solutions are obtained and shown to perform close to the optimal solution.

Index Terms—Decision-making, power allocation, imperfect CSI, sensor communications.

I. INTRODUCTION

A SENSOR network typically consists of a large number of sensor nodes, and a single sink node which acts as a data collection point of the sensor network. Sensor nodes are usually battery-powered. Thus, all operations of the sensor nodes, including the transceiver operations, should be carefully managed to ensure a long operational life span [1], [2].

Much effort has been spent in developing efficient power control algorithms to improve system performance even with imperfect channel state information (CSI) [3]–[5]. In all these works, the availability of CSI forms the basis of efficient power control schemes that allow a reduction in the transmit energy. Nevertheless, the sensor node uses some receive energy in the process of obtaining the CSI, which is typically imperfect in practice. The receive energy is comparable to the transmit energy, especially for sensors [6], [7], and cannot be ignored. Given a choice, the sensor node may even choose not to obtain the imperfect CSI, as the receive energy used to acquire the CSI can be higher than the expected reduction in transmit energy resulting from the availability of the CSI.

To study the net power saving of obtaining CSI, we consider a wireless sensor network, where a sensor is communicating with a data sink at a target data rate subject to a target outage probability. The data sink sends a CSI packet to the sensor, from which the sensor node can choose to listen and extract the CSI. We focus on the extreme case in which the packet conveys a one-bit (imperfect) CSI to the sensor node. If the sensor decides to listen to the CSI packet, it obtains the CSI by using up some receive energy but may use less energy to satisfy the outage probability constraint; otherwise the sensor node has no CSI and may use higher power for data transmission.

In this paper, we design the optimal transmit strategy for the sensor node, consisting of the joint decision of whether to listen to the CSI and the corresponding power allocation, so that the expected total transmit and receive energy is minimized. The key insight that can be drawn is that we should empower sensors to decide whether to receive the CSI, especially when the CSI is imperfect which results in significant uncertainty. Numerical results confirm that the optimal decision and corresponding power allocation result in significant power saving compared to fixed or naive schemes. Surprisingly, the performance is also close to the case when full CSI is available. We also obtain closed-form expressions of the suboptimal solutions which perform close to the optimal ones in our numerical studies.

II. SYSTEM MODEL

We consider a wireless sensor network scenario, in which a sensor node collects data which is to be communicated to a data sink at a target data rate $R$ in bits per channel use. The received signal at the data sink is modeled as

$$y = \sqrt{P}h x + n$$

(1)

where $x$ denotes the transmitted signal with $\mathbb{E}[|x|^2] = 1$, $\mathbb{E}[-]$ denotes the expectation operator, $P$ is the transmit power, $h$ is the instantaneous channel gain, and $n$ is the additive zero-mean white circularly complex Gaussian variable with unit variance. We assume that $h$ is Rayleigh distributed with $\mathbb{E}[|h|^2] = \bar{h}$.

The channel is typically fast fading, however, we assume the variance of the channel gain $\bar{h}$ is known to the sensor node, which is a long-term statistical parameter that can be measured given sufficient time. To aid the sensor node, the data sink transmits packets containing one bit of CSI to the sensor, indicating either $|h|^2 \leq \bar{h}$ or $|h|^2 > \bar{h}$, where $\gamma_h$ is a predetermined threshold known by the sensor. We assume that the sensor node can obtain the one bit of CSI perfectly.

We use power and energy interchangeably in this paper. The sensor node makes decisions as follows, see also Fig. 1.

- If the sensor node chooses to ignore the CSI packet, it will use power $P = P_0$ for data transmission.
subject to a target outage probability of $\bar{\gamma}$.

\[ \bar{\gamma} = \frac{\ln(1 - \gamma)}{\ln(1 + \gamma)} \]

The optimization problem can appeartractable as it is not a convex optimization problem. Nevertheless, the solution can be obtained by solving a single-variable optimization problem.

We split $S = \bigcup_{i=1}^{4}$ into four disjoint spaces: $S_1 = \{(P_1, P_2) \in S : P_1, P_2 < Q(R)\}$, $S_2 = \{(P_1, P_2) \in S : P_1 < Q(R), P_2 \geq Q(R)\}$, $S_3 = \{(P_1, P_2) \in S : P_1, P_2 \geq Q(R)\}$, $S_4 = \{(P_1, P_2) \in S : P_1 \geq Q(R), P_2 < Q(R)\}$, where $Q(R) = \left(2^{R} - 1\right)/\gamma_0$. We solve (3) by restricting the variable space to $S_i$ for $i = 1, \ldots, 4$.

**Optimization over $S_1$ (Case $S_1$):** Then (3) simplifies as $\min_{(P_1, P_2) \in S} P_{\text{rx}}$, s.t. $P_1 < Q(R)$, $P_2 < Q(R)$, $P_r \left(|h|^2 \leq \gamma_0\right) + P_r \left(|h|^2 \geq \left(2^{R} - 1\right)/P_1\right) \leq \alpha$. If $\gamma_0 > \gamma_0 \ln(1 - \gamma)$, then $P_r \left(|h|^2 \leq \gamma_0\right) > \alpha$ and so there is no feasible solution; otherwise, the optimal solution is obtained as $P_1^* = 0$, $P_2^* = \left(2^{R} - 1\right)/\gamma_0$.

**Optimization over $S_2$ (Case $S_2$):** Then (3) simplifies as $\min_{(P_1, P_2) \in S} P_{\text{rx}}$, s.t. $P_1 < Q(R)$, $P_2 \geq Q(R)$, $P_r \left(|h|^2 \leq \gamma_0\right) \leq \alpha$. If $\gamma_0 > \gamma_0 \ln(1 - \gamma)$, then $P_r \left(|h|^2 \leq \gamma_0\right) > \alpha$ and there is no feasible solution; otherwise, the optimal solution is obtained as $P_1^* = 0$, $P_2^* = Q(R)$.

**Optimization over $S_3$ (Case $S_3$):** Then (3) simplifies as $\min_{(P_1, P_2) \in S} P_{\text{rx}}$, s.t. $P_1 \geq Q(R)$, $P_2 \geq Q(R)$, $P_r \left(|h|^2 \left(2^{R} - 1\right)/P_1\right) \leq \alpha$. If $\gamma_0 \leq \gamma_0 \ln(1 - \gamma)$, there will be no power saving compared to case of no CSI because $P_1, P_2 \geq Q(R) \geq \left(2^{R} - 1\right)/\gamma_0 = P_2^*$; otherwise, the optimal solution is obtained as $P_1^* = \left(2^{R} - 1\right)/\gamma_0$, $P_2^* = Q(R)$.

**Optimization over $S_4$ (Case $S_4$):** Then (3) simplifies as $\min_{P_1 \geq P_2 \geq Q(R)} P_{\text{rx}}$.

This optimization problem is non-convex because the constraint in (4) is a non-convex set. Nevertheless, the optimal solution can be obtained with the help of Claim 1 and Claim 2.

**Claim 1:** Suppose that the optimal $P_2$ in the optimization problem (4) satisfies $P_2 < Q(R)$. Let $Q_1$ be the solution of $Q(R) = P_1 + Q_2$ be the solution of (6) with $P_2 = 0$ and the inequality set as equality, then the optimal $P_1^*$ is given by $\max(Q_1, Q_2)$ if $\alpha > \exp(-\gamma_0/\gamma)$. The objective is to minimize $P_1$ if $P_2 = 0$.

**Proof:** Since the objective is to minimize $P_1$ if $P_2 = 0$, we should choose the smallest $P_1$ such that constraint (5) or (6) is tight. If $\alpha \leq \exp(-\gamma_0/\gamma)$, then $e \left(2^{R} - 1\right)/\gamma_0 + e \left(2^{R} - 1\right)/\gamma_0 \geq 1 + e^{-\gamma_0/\gamma} - \alpha$. This optimization problem is non-convex because the constraint in (4) is a non-convex set. Nevertheless, the optimal solution can be obtained with the help of Claim 1 and Claim 2.

For Rayleigh fading channels, we obtain $Q_1 = Q(R)$ and $Q_2 = \frac{\gamma_0 \ln(1 + \exp(-\gamma_0/\gamma))}{\gamma_0 \ln(1 + \exp(-\gamma_0/\gamma)) - \alpha}$. As $\exp(-\gamma_0/\gamma) < \exp(-\gamma_0/\gamma) + 1 - \alpha$, we have $\gamma_0 > \gamma_0 \ln(1 + \exp(-\gamma_0/\gamma) - \alpha)$. Thus, $\max(Q_1, Q_2) = Q_2$. That is, the constraint (5) is not active.
TABLE I

<table>
<thead>
<tr>
<th>Case</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\gamma_n \leq -\gamma \ln(1-\alpha)$</td>
</tr>
<tr>
<td>2</td>
<td>$\gamma_n &gt; -\gamma \ln(1-\alpha)$</td>
</tr>
</tbody>
</table>

Global Optimal Solution

Suboptimal Solution

Claim 2: Suppose that the optimal $P_2$ in the optimization problem $P_2^*$ satisfies $P_2^* > 0$. Then the optimal $P_1^*, P_2^*$ satisfy constraint (6) with equality.

Proof: Suppose the optimal $P_1^*, P_2^*$ do not satisfy constraint (6) with equality, i.e. the inequality is strict. Then it is always possible to reduce $P_2$ by an infinitely small amount to give a smaller value for the objective function, which satisfies constraints (5) and (6), thus contradicting its optimality.

Suppose $P_2^* > 0$. Since (6) is obtained as an equality, we can substitute $P_1$ as a function of $P_2$ into (4), i.e., $P_1 = f(P_2) \triangleq \frac{\gamma \ln[1-\exp(-\gamma \bar{\gamma} / \gamma)]-\alpha-\exp(-\gamma \bar{\gamma} / \gamma)}{\gamma \ln(1-\alpha)}$. Then we can solve a single-variable optimization problem, subject to the constraint of (5) only. Since $P_1 = f(P_2) \geq Q(R)$ in Case $S_3$, after some algebraic manipulations we get $P_2 \leq \frac{\gamma \ln(1-\alpha)}{\gamma \ln[1-\exp(-\gamma \bar{\gamma} / \gamma)]-\alpha-\exp(-\gamma \bar{\gamma} / \gamma)}<0$; this leads to $P_2 \geq Q_3$, where

$$Q_3 \triangleq \begin{cases} \frac{\gamma \ln[\exp(-\gamma \bar{\gamma} / \gamma)-\alpha]}{\gamma \ln(1-\alpha)} & \text{if } \alpha < \exp(-\gamma \bar{\gamma} / \gamma) \\ 0 & \text{otherwise} \end{cases}$$

In summary, it is sufficient to solve the single-variable optimization problem for $P_2$

$$\begin{align*}
& \text{min}_{P_2} (1-\exp(-\gamma \bar{\gamma} / \gamma)) f(P_2) + \exp(-\gamma \bar{\gamma} / \gamma) P_2 \\
& \text{s.t. } Q_3 \leq P_2 \leq \min \left[ Q(R), \frac{-(2^R-1)}{\gamma \ln(1-\alpha)} \right] 
\end{align*}$$

where $Q_3$ is given by (7).

Global optimization over $S$ (Combining all Cases): By comparing the optimal solutions for all cases above and choosing the one with the smallest power, we obtain the global optimal solution. Without loss of generality, we do not consider the cases with no feasible solution, nor the case when there is no power saving compared to the case of no CSI (as the final goal is to listen to CSI only if it is beneficial). Moreover, it can be shown that the expected transmit power in Case $S_1$ is always less than that of Case $S_2$ (we omit the proof); hence we exclude Case $S_2$ subsequently. We summarize the optimal solutions of the remaining cases in Table I.

C. Optimization of $\gamma_n$

To further reduce the total expected transmit power, we optimize the threshold $\gamma_n$ in this subsection. Optimizing $\gamma_n$ directly based on the global optimal solution in Table I is not tractable, as no closed-form expression is provided for Case $S_4$. A suboptimal approach is to ignore Case $S_4$ in the optimization problem of (3). We call the combination of the solutions of Cases $S_1$, $S_2$ and $S_3$ the suboptimal solution, as shown in Table I. From simulation results in Section V, the suboptimal solution has very close performance to the optimal solution. Thus, we will optimize $\gamma_n$ based on the suboptimal solution.

The optimization problem for $\gamma_n$ to minimize the total expected transmit power is formulated as

$$\min_{\gamma_n \in B} \hat{P}_{\text{tx}} \quad \text{(9)}$$

where $\hat{P}_{\text{tx}}$ denotes the minimum power obtained from the suboptimal solution in Table I, given by

$$\hat{P}_{\text{tx}} = \begin{cases} \frac{-(2^R-1)e^{-\gamma_n / \gamma}}{\gamma \ln(1-\alpha)} - \frac{(2^R-1)(1-e^{-\gamma_n / \gamma}) - (2^R-1)e^{-\gamma_n / \gamma}}{\gamma \ln(1-\alpha)} \gamma_n, \quad \text{for } \gamma_n \in B_1 \\ 0, \quad \text{for } \gamma_n \in B_2 
\end{cases}$$

and $B_1$, $B_2$, $B$ are the variable spaces defined as $B_1 = \{\gamma_n \in B : \gamma_n \leq -\gamma \ln(1-\alpha)\}$, $B_2 = \{\gamma_n \in B : \gamma_n > -\gamma \ln(1-\alpha)\}$ and $B = \{\gamma_n : \gamma_n \geq 0\}$, respectively. Clearly, $B = U_1 B_1$.

Next, we solve (9) by restricting the variable to be in $B_1$ separately. To obtain the global solution, we compare the two optimal solutions and choose the one that minimizes $\hat{P}_{\text{tx}}$.

Claim 3: The optimal solution for the optimization problem (9) in the case of $\gamma_n \in B_1$ is given by $\gamma_{n,1} = -\gamma \ln(1-\alpha)$.

Proof: Since the objective function in the case of $\gamma_n \in B_1$ denoted as $G_1(\gamma_n) = \frac{-(2^R-1)e^{-\gamma_n / \gamma}}{\gamma \ln(1-\alpha)}$, is decreasing in $\gamma_n$, the optimal solution is obtained as $\gamma_{n,1} = -\gamma \ln(1-\alpha)$.

Claim 4: The optimal solution for the optimization problem (9) in the case of $\gamma_n \in B_2$ is given by $\gamma_{n,2} = \max \{-\gamma \ln(1-\alpha) + \epsilon, \gamma_{n,1}\}$, where $\epsilon > 0$ approaches zero.

Proof: To find the optimal $\gamma_{n,2}$, we first remove the constraint $\gamma_n > -\gamma \ln(1-\alpha)$ and solve an unconstrained optimization problem. Equating the first derivative of the objective function $G_2(\gamma_n) = -\gamma \ln(1-\alpha) + (2^R-1) \exp(-\gamma_n / \gamma)) / \gamma \ln(1-\alpha) + (2^R-1) / \gamma \exp(-\gamma_n / \gamma)) / \gamma \ln(1-\alpha)$ with respect to $\gamma_n$ to zero gives $\gamma_{n,2} = \gamma_{n,1}/\gamma \ln(1-\alpha)$. In fact, the first derivative of the objective function is positive if and only if $\gamma_n > \gamma_{n,2}$, which indicates that the optimization problem is quasi-convex [8]. Thus, the optimal solution for the optimization problem is $\gamma_{n,2} = \max \{-\gamma \ln(1-\alpha) + \epsilon, \gamma_{n,1}\}$.

Global optimization over $B$ (Combining all Cases): To obtain the global optimal solution for $\gamma_n$, we need to compare $G_1(\gamma_{n,1})$ and $G_2(\gamma_{n,2})$. If $G_1(\gamma_{n,1}) < G_2(\gamma_{n,2})$, the global optimal solution is $\gamma_{n} = \gamma_{n,2}$; otherwise, it is $\gamma_{n} = \gamma_{n,1}$.

D. Decision-making and Power Allocation Strategy

The availability of CSI forms the basis of an efficient power control scheme that allows a reduction in the transmit energy. The power saving with one bit of CSI compared to the case of no CSI is given by $\Delta = 10 \log_{10} \left[ P_0 - \left(1 - \exp(-\gamma_\text{th} / \gamma))P_1 + \exp(-\gamma_\text{th} / \gamma))P_2 \right] \right]$ dB. However, to obtain the CSI, which is typically imperfect in practice, requires that some receive energy that is consumed

1It can be shown analytically that if $\alpha$ is small or $\gamma_\text{th}$ is large enough, the optimal solution in Case $S_4$ performs better than the neighboring feasible solutions in Case $S_4$, which suggests that Case $S_4$ can be ignored without significant loss in performance.
in the process of obtaining the CSI. If $\Delta < P_{rx}$ dB, the receive energy spent is higher than the expected reduction in transmit energy resulting from the availability of the CSI. If $\Delta \geq P_{rx}$ dB, listening to the CSI packet results in a reduction of the total energy with our proposed power control scheme. To perform this tradeoff systematically, the following decision-making and power allocation steps are performed by the sensor node:

Step 1: The sensor node calculates $\gamma_0^*, P_0^*$, $P_1^*$ and $P_2^*$ with the given target outage requirement $\alpha$ and channel variance $\gamma$.

Step 2: The sensor node compares the difference of the expected transmit power with and without one-bit CSI, i.e., $\Delta$.

Step 3: If $\Delta < P_{rx}$ dB, the sensor node ignores the CSI packet and transmits with power $P_0^*$; otherwise, go to Step 4.

Step 4: If $\Delta \geq P_{rx}$ dB, the sensor node listens to the CSI packet. If $\gamma > \gamma_0^*$, the sensor node transmits with power $P_2^*$; otherwise, it transmits with power $P_1^*$ for data transmission.

Thus, the sensor node will only need to calculate $\gamma_0^*, P_0^*, P_1^*, P_2^*, \Delta$ and make the decision of listening to CSI or not once for a fixed variance of the channel. After making the decision, it will keep ignoring or listening to the CSI packet.

V. SIMULATION RESULTS

We perform simulations to show the performance of the proposed power control schemes. We assume $\gamma = 1$. For comparison, we also include the performance when full CSI is available. In this case, the sensor is still allowed to decide whether to send the packet (in which case it uses just enough energy such that the packet is received correctly, e.g., $P_{tx} = \frac{2^{\alpha^*} - 1}{\alpha^*}$) or not to send (in which case it decides too much energy is needed and so it rather saves the energy by not sending at all, e.g., $P_{tx} = 0$ for $|h|^2 < -\ln(1-\alpha)$). Note that in the figures the solid lines show the power saving of the global optimal solution with one-bit CSI compared to no CSI cases; the curves with marker x show the suboptimal solution with one-bit CSI compared to no CSI cases; the dash lines show the power saving with full CSI compared to no CSI cases.

In Fig. 2, we show how the power saving $\Delta$ with the optimized $\gamma_0^*$ changes with target rates $R$. We can see that $\Delta$ is significant, which indicates that our proposed power control with imperfect CSI leads to significant power saving compared to the case of no CSI. In fact, the threshold is independent of the target rate. Also, we see that the suboptimal solution achieves a very close performance to the optimal solution. Thus, in practice we can simply adopt the suboptimal solution with closed-form expressions which is less complex.

Fig. 3 shows how the power saving $\Delta$ changes with $\gamma_0^*$ under various outage constraints. The square markers are the power saving with the optimal thresholds $\gamma_0^*$ derived in Section IV.C. Although these thresholds were derived based on the suboptimal power allocation solution in Table I, they lead to close to the maximal possible power saving based on the global optimal power allocation solution in Table I. Thus, adopting the suboptimal power allocation solution is sufficient in practice. From both figures, we see that our proposed one-bit CSI feedback protocol also achieves very close performance to the full CSI case if the threshold $\gamma_0^*$ is optimized.

VI. CONCLUSIONS

In the existing literature, typically power control is considered assuming no cost is incurred in obtaining the CSI. In this paper, we explicitly accounted for this cost. Moreover, we have empowered sensors to acquire CSI only when it saves the overall power. This is a step towards realizing intelligent, self-adapting wireless sensor networks by incorporating aspects of physical layer processing in the design.

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