Linear Analog Network-Coded OFDM in Two-way Relay Channels

Nicholas Heng-Loong Wong, Sumei Sun, and Chin Keong Ho
Institute for Infocomm Research, Agency for Science, Technology, and Research
1 Fusionopolis Way, #21-01 Connexis, Singapore 138632
Email: nicholas_whl@hotmail.com, {sunsm, hock}@i2r.a-star.edu.sg

Abstract—We study analog network-coded (ANC) orthogonal frequency division multiplexing (OFDM) in two-way relay channels and propose three linear analog network coding (LANC) schemes. Motivated by the fact that higher frequency diversity has been made available in the frequency-selective ANC two-way relay channels, the proposed LANC is applied to the frequency-domain OFDM signals with unitary matrices. The three LANC schemes are differentiated by the nodes at which they are implemented, namely, at the relay only, at the sources only, and at both the sources and the relay. Compared with the conventional ANC scheme, the proposed LANC schemes can exploit the frequency diversity more effectively. While the relay-only LANC provides only marginal performance improvement, significant performance gains are achieved by the source-only and source-relay LANC schemes.

I. INTRODUCTION

Physical layer network-coding has seen significant development in recent years. It seeks in essence to increase the capacity of wireless systems by cleverly making use of the broadcast nature of wireless channels as well as the inevitable interference and noise effects which they carry [1]. Network-coding is particularly useful in relay networks, the simplest scenario of which is the 3-node setup as depicted in Fig. 1. When the two nodes A and B are too far apart, they can exchange information with each other with the help of an intermediary relay node R. Supposing each message signal is one symbol interval in length, the conventional approach applying no network coding takes a total of four such intervals for nodes A and B to exchange messages s_A and s_B respectively, as illustrated in Fig. 1a. The efficiency of such an approach is therefore very low.

One simple method utilizing network-coding reduces the number of required symbol intervals from 4 to 3 as shown in Fig. 1b. In this scheme, A and B take turns to send their signal to R in the first two symbol intervals. R detects the signals and then in the third symbol interval broadcasts the XOR-ed version of the two signals, i.e., s_A⊕s_B, to both nodes simultaneously. Each node processes the received combination to obtain the destined messages. In [1], this scheme is referred to as Digital Network-coding (DNC).

By making use of the broadcast nature of wireless transmission, a two-symbol interval method was proposed in [1] to push the limits further. As illustrated in Fig. 1c, in this scheme, both A and B transmit their own messages to R simultaneously in the first symbol interval, also known as the multiple-access (MAC) phase [2]. What R receives is thus a combination of s_A and s_B. R then scales the power of the received signal and broadcasts it to A and B in the second symbol interval, the broadcast phase [2]. Finally, A and B process their respective received signals to decipher the intended messages. This scheme is referred to as Analog Network-coding (ANC) [1] or Physical-layer Network-coding (PNC) [3].

In this paper, we consider multipath fading and focus our study on ANC design for orthogonal frequency division multiplexing (OFDM) signals [4]. With the multipath interference being mitigated by OFDM signaling, the above ANC scheme can be applied to OFDM in a very straightforward manner. However, considering the fact that the effective channel in such an ANC two-way relay network is the product of the two individual source—relay channels, the multipath order is therefore increased. As a result, the potential frequency diversity gain that can be exploited is increased. However, such aspect has not been well explored in the literature, to the best of our knowledge. In [2], it was proposed to apply tone permutation to the received signal at the relay node, and power allocation was optimized to maximize the system’s sum rate. The diversity gain, however, was not investigated.

In [5] and [6], linear pre-transform (LPT) was proposed to exploit the frequency diversity of OFDM in frequency selective channels. Motivated by the good results in [5] and [6], we propose to exploit the frequency diversity gain in the two-way relay channels through linear analog network coding (LANC). The LANC is implemented with unitary matrices and applied to the frequency-domain OFDM signals. Three schemes are considered, namely, LANC at the relay node (LANC-R), LANC at the source nodes (LANC-S), and LANC at both the source and relay nodes (LANC-SR). In particular, linear coding using a phase-rotated discrete Fourier transform (DFT) matrix, as suggested in [5] and [6] is used in LANC-R, LANC-S, and at the source nodes in LANC-SR, and subcarrier permutation is implemented at the relay node in LANC-SR. The bit error rate (BER) performance of these schemes was evaluated using the maximum-likelihood-achieving sphere decoding algorithm [7]. For the OFDM signals, we assume that a cyclic prefix, of length at least the total number of multipath taps of the equivalent channel, is appended before transmission at the sources and then removed after reception at the sources.
Our results show that the proposed LANC-S and LANC-SR provide significant performance improvement over the conventional ANC OFDM systems, which are mainly from the much higher frequency diversity order achieved in the proposed systems. For the LANC-R, however, the considered precoding design is not able to effectively exploit the frequency diversity gains, hence only marginal performance improvement has been observed.

In the rest of the paper, we turn to the frequency domain representation for the signals. Frequency domain variables are represented by uppercase letters, whereas those in the time domain are in lowercase. That is, $s_X = s_X[n]$ is a frequency-time domain pair for the time domain signal $s_X$. Variables that are column vectors or matrices are written in boldface notation, while those in italicized roman are scalars. $S_{X_k}$ represents the $k$th row of the matrix or column vector $S_X$. The superscript $^H$ is used to denote the Hermitian of a matrix. When describing the traversal of a signal from a node/relay, say $X$ to a destination, say $Y$, the following notation is used: $X \rightarrow Y$.

The paper is organized as follows. Section II gives an overview of the conventional ANC OFDM scheme in two-way relay channels, and develops the signal model. In Section III, we describe the three proposed LANC schemes. The simulated performance results are presented in Section IV, and finally the conclusion is given in Section V.

II. CONVENTIONAL ANC-OFDM IN TWO-WAY RELAY CHANNELS

The conventional point-to-point OFDM signal can be modeled as

$$Y = HS + \nu$$

where $H$ is the diagonal channel matrix with its diagonal elements representing the channel coefficients of the subcarriers, $S$ is the transmitted signal, and $\nu$ is the additive-white-gaussian-noise (AWGN) at the receiver.

In the two-symbol interval (two-time-slot) ANC-OFDM two-way relay, the following signal is received by the relay $R$ during the first time slot, i.e., the MAC phase:

$$Y_R = \frac{1}{\sqrt{2}}H_A S_A + \frac{1}{\sqrt{2}}H_B S_B + \nu_R$$

where $H_A$ and $H_B$ denote the $A \rightarrow R$ and $B \rightarrow R$ channels, respectively, $S_A$ and $S_B$ are the (vector) signals sent from $A$ and $B$ respectively, and $\nu_R$ is the AWGN at relay $R$ with zero mean and variance $\sigma^2_R$. The presence of the $\frac{1}{\sqrt{2}}$ coefficients normalizes the total transmit signal power to $\frac{1}{2}$ with the assumption that the source signal power is unity, i.e., $E[|S_A|^2] = E[|S_B|^2] = 1$, and $E[\cdot]$ denotes the expected value.

The relay in the conventional ANC scheme will scale the power of the received signal $Y_R$, say to 1, before transmitting to the two source nodes, i.e.,

$$S_R = \sqrt{\gamma}Y_R$$

where $\gamma$ is calculated as

$$\gamma = \frac{1}{E[|Y_R|^2]}
\begin{align*}
&= \frac{2}{E[\frac{1}{N}\sum_{k=1}^{N}|H_{A_k}S_A|^2 + \sum_{k=1}^{N}|H_{B_k}S_B|^2] + 2\sigma^2_R} \\
&= \frac{1}{1 + \sigma^2_R}
\end{align*}$$

and we have assumed that the channel has unit gain.

Assuming a reciprocal channel, i.e., $H_{A(R\rightarrow A)} = H_{A(A\rightarrow R)}$ and $H_{B(R\rightarrow B)} = H_{B(B\rightarrow R)}$, we have the received signal by node $A$ according to (1) and (3),

$$Y_A = H_A S_R + \nu_A$$

$$= \sqrt{\frac{\gamma}{2}}(-H_A^2 S_A + H_A H_B S_B) + \sqrt{\gamma}H_A \nu_R + \nu_A$$

where $\nu_A$ denotes the AWGN at node $A$ with zero mean and variance $\sigma^2_A$, and the received signal by node $B$ as

$$Y_B = \sqrt{\frac{\gamma}{2}}(-H_B H_A S_A + H_B^2 S_B) + \sqrt{\gamma}H_B \nu_R + \nu_B$$

where $\nu_B$ denotes the AWGN at node $B$ with zero mean and variance $\sigma^2_B$.

Assuming the channel state information (CSI) is available at $A$ and $B$, self-interference cancellation can be performed and the signals used for detection are respectively,

$$\hat{Y}_A = \sqrt{\frac{\gamma}{2}}H_A H_B S_B + \sqrt{\gamma}H_A \nu_R + \nu_A$$

$$\hat{Y}_B = \sqrt{\frac{\gamma}{2}}H_A H_B S_B + \nu_A$$

where $\hat{Y}_A$ and $\hat{Y}_B$ are the output signals of the adaptive cancellers at node $A$ and $B$, respectively.

Fig. 1. Message transactions in a two-way relay system, taking (a) 4 symbol intervals (no network-coding), (b) 3 symbol intervals and (c) 2 symbol intervals. $t$ represents the symbol interval index.
for node $A$, and
\[
\tilde{Y}_B = \sqrt{2}H_BH_AES_A + \sqrt{\gamma}H_B\nu_R + \nu_B
\]
for node $B$.

Under the assumption that $H_A$ and $H_B$ are known to $A$ and $B$, the noise terms $\tilde{\nu}_A$ and $\tilde{\nu}_B$ are still Gaussian, and they have the following statistics:
\[
\begin{align*}
E[\tilde{\nu}_A] &= 0, \\
E[\tilde{\nu}_A\tilde{\nu}_A^H] &= \gamma\sigma^2_B H_A H_A^H + \sigma^2_A I, \\
E[\tilde{\nu}_B] &= 0, \\
E[\tilde{\nu}_B\tilde{\nu}_B^H] &= \gamma\sigma^2_B H_B H_B^H + \sigma^2_B I
\end{align*}
\] (9)
where $I$ is the identity matrix of appropriate dimension. Maximum likelihood detection (MLD) can therefore be performed as
\[
\begin{align*}
\hat{S}_B &= \arg\max_{\{S_B\}} \Pr(\tilde{Y}_A|S_B; \sqrt{2}H_AH_BS_A, H_A) \\
\hat{S}_A &= \arg\max_{\{S_A\}} \Pr(\tilde{Y}_B|S_A; \sqrt{2}H_BH_AS_A, H_B)
\end{align*}
\] (10)
to get the signals $\tilde{S}_B$ and $\tilde{S}_A$ intended for $A$ and $B$ respectively.

III. PROPOSED LANC SCHEMES FOR TWO-WAY RELAY OFDM CHANNELS

From (7) and (8), we can easily see that the ANC-OFDM signals experience a product channel,
\[H_{\text{prod}} = H_AH_B = H_BH_A.\]
The equivalent time domain channel is the linear convolution of the time domain multipath channels, i.e.,
\[h_{\text{prod}} = h_A \otimes h_B\]
where $\otimes$ denotes the Kronecker product operation, with the total number of multipaths given as
\[L_{\text{prod}} = L_A + L_B - 1\]

Next, we present the three LANC designs. They are distinguished by the nodes at which LANC is applied, namely, LANC-S if it is applied only at the source nodes, LANC-R if only at the relay, or LANC-SR if it is implemented at both the source and the relay nodes.

A. LANC-R

In this scheme, LANC by a unitary matrix $W$ is applied at the relay received signal $Y_R$, as
\[S_R = \sqrt{\gamma}WY_R.\] (11)
The respective source nodes therefore receive, according to (7), (8) and (11),
\[
\begin{align*}
\tilde{Y}_A &= \sqrt{\gamma}H_AWHS_B + \sqrt{\gamma}H_AW\nu_R + [\nu_A] \\
\tilde{Y}_B &= \sqrt{\gamma}H_BWHS_A + \sqrt{\gamma}H_BW\nu_R + [\nu_B]
\end{align*}
\] (12)
after self-interference removal.

Noting that $W$ is a unitary matrix, $W\nu_R$ is therefore still AWGN with zero mean and variance $\sigma^2_R$, and $\tilde{\nu}_A$ and $\tilde{\nu}_B$ are Gaussian with the same statistical properties as given in (9). The MLD at nodes $A$ and $B$ are therefore given as
\[
\begin{align*}
\hat{S}_B &= \arg\max_{\{S_B\}} \Pr(\tilde{Y}_A|S_B; \sqrt{2}H_AWHS_B, H_A) \\
\hat{S}_A &= \arg\max_{\{S_A\}} \Pr(\tilde{Y}_B|S_A; \sqrt{2}H_BWHS_A, H_B)
\end{align*}
\] (13)

B. LANC-S

In this scheme, the messages are precoded at the source nodes with the mutually known $W$ matrix. This means that sources $A$ and $B$ send out $WS_A$ and $WS_B$, respectively, giving rise to the received signal at $R$:
\[Y_R = \frac{1}{\sqrt{2}}H_AWS_A + \frac{1}{\sqrt{2}}H_BWS_B + \nu_R\] (14)
according to (2).

Following this, $R$ performs the simple amplify-and-forward operation in (3) and broadcasts the resulting $S_R$. $A$ and $B$ then receive the broadcast through their own channels, and obtain the following signal after self-interference cancellation
\[
\begin{align*}
\tilde{Y}_A &= \sqrt{\gamma}H_AWHS_B + \sqrt{\gamma}H_AW\nu_R + [\nu_A] \\
\tilde{Y}_B &= \sqrt{\gamma}H_BWHS_A + \sqrt{\gamma}H_BW\nu_R + [\nu_B]
\end{align*}
\] (15)
respectively, and $\tilde{\nu}_A$ and $\tilde{\nu}_B$ have the same statistical properties as given in (9).

The sources then get
\[
\begin{align*}
\hat{S}_B &= \arg\max_{\{S_B\}} \Pr(\tilde{Y}_A|S_B; \sqrt{2}H_AWHS_B, H_A) \\
\hat{S}_A &= \arg\max_{\{S_A\}} \Pr(\tilde{Y}_B|S_A; \sqrt{2}H_BWHS_A, H_B)
\end{align*}
\] (16)
through MLD.
C. LANC-SR

As suggested by the name, LANC is applied at both the source and relay nodes in this scheme. The process starts off in the same way as in LANC-S where the sources send out \( WS_\text{A} \) and \( WS_\text{B} \), and \( R \) receives the interfered signal according to (14).

At the relay however, in addition to performing power normalization, another unitary linear precoding is applied to \( Y_\text{R} \). Considered in this paper in particular is a permutation matrix \( P \) which permutes the received signal at different subcarriers so as to decorrelate the adjacent subcarriers and improve the diversity performance. Therefore, the signal that is broadcasted is

\[
S_\text{R} = \sqrt{\gamma} PY_\text{R}. \tag{17}
\]

Finally, \( A \) and \( B \) receive the broadcast and after self-interference removal, they obtain respectively,

\[
\begin{align*}
\tilde{Y}_\text{A} & = \sqrt{\frac{\gamma}{2}} [H_\text{A}PH_\text{B}WS_\text{B}] + \frac{\gamma}{2} [H_\text{A}H_\text{B}] P_\nu_\text{R} R + [\nu_\text{A} \nu_\text{B}] \\
\tilde{Y}_\text{B} & = \sqrt{\frac{\gamma}{2}} [H_\text{A}PH_\text{B}WS_\text{B}] + \frac{\gamma}{2} [H_\text{A}H_\text{B}] P_\nu_\text{R} \tilde{R} + [\nu_\text{A} \nu_\text{B}] \tag{18}
\end{align*}
\]

Again \( \nu_\text{A} \) and \( \nu_\text{B} \) have the same statistical properties as given in (9).

The MLD is therefore performed at \( A \) and \( B \) as in

\[
\begin{align*}
\hat{S}_\text{B} & = \text{arg} \max_{\{S_\text{B}\}} \text{Pr} \left( \tilde{Y}_\text{B} | S_\text{B} : \sqrt{\frac{\gamma}{2}} H_\text{A}PH_\text{B}W, H_\text{B} \right) \\
\hat{S}_\text{A} & = \text{arg} \max_{\{S_\text{A}\}} \text{Pr} \left( \tilde{Y}_\text{A} | S_\text{A} : \sqrt{\frac{\gamma}{2}} H_\text{A}PH_\text{B}W, H_\text{B} \right) \tag{19}
\end{align*}
\]

D. Remarks

Comparing the three proposed schemes, we notice that the Gaussian noise characteristics are the same. As suggested by the names, they differentiate from each other by the nodes at which they are applied, or more specifically, by the channel to which they are applied. The LANC-S introduces the linear transformation to the product channel \( H_\text{A}H_\text{B} \), hence it is able to exploit a maximum of \( L_{\text{prod}} \)-th order frequency diversity with either the MLD [6], or the iterative subcarrier reconstruction receiver [5]. In LANC-SR, the permutation at the relay node changes the power delay profile of the partial channel \( H_\text{A} \) and \( H_\text{B} \) (hence the product channel as well), but the total number of the multipaths in the product channel maintains the same. With the linear precoding performed at the source nodes, it is also able to exploit the maximum frequency diversity order of \( L_{\text{prod}} \). In LANC-R, the linear transform works on only the partial channel \( H_\text{A} \) or \( H_\text{B} \). Therefore, it is predictable that the performance of LANC-R is inferior to the other two schemes.

IV. SIMULATION RESULTS

In this section, we present the simulation results of our proposed LANC schemes. In the OFDM system, we have \( N = 16 \) subcarriers and the cyclic prefix length is set to \( CP = 4 \). Quadrature Phase Shift Keying (QPSK) signals are adopted. For the time domain channels \( h_\text{A} \) and \( h_\text{B} \), we consider uniform power delay profiles with \( L_\text{A} = L_\text{B} = 4 \) independent and identically distributed (i.i.d.) complex Gaussian multipath components. Perfect self-interference cancellation is assumed at the source nodes before the MLD-achieving sphere decoder [7] is used to obtain the decoded information sequence.

The linear precoding matrix \( W \) used in simulations is an \( N \)-point discrete Fourier transform (DFT) phase rotation matrix given by:

\[
W = \frac{F_N}{\sqrt{N}} \text{Diag} \left( 1, e^{-j\frac{2\pi}{N}}, \ldots, e^{-j\frac{2\pi(N-1)}{N}} \right) \tag{20}
\]

where \( F_N \) denotes the \( N \times N \) DFT matrix with element \( f_{m,n} = \exp \left( -j\frac{2\pi (m-1)(n-1)}{N} \right) \), \( m, n = 1, 2, \ldots, N \), and the permutation matrix \( P \) in the LANC-R scheme has a random permutation order.

We depict the BER versus signal-to-noise-ratio (SNR) performance of the proposed schemes in Fig. 2 for the \( L_\text{A} = L_\text{B} = 4 \) channels. Also included in the figure is the performance of the conventional ANC scheme. We can clearly see that all the proposed schemes introduce performance improvement over the conventional ANC scheme. Among the three proposed schemes, LANC-SR has the best performance and the highest diversity order. At an SNR of 18 dB, compared to the ANC scheme which has a BER of \( 5.028 \times 10^{-2} \), the LANC-SR scheme achieves a significantly lower BER of \( 4.733 \times 10^{-5} \). The LANC-S scheme has slightly worse performance and slightly lower frequency diversity than the LANC-SR, whereas the LANC-R introduces the least performance improvement. This has been expected as we have briefly discussed in Section III-D. The LANC-R can only affect a partial channel, whereas LANC-S and LANC-SR work on the product channel, and hence have the capability of exploiting full frequency diversity order.
V. CONCLUSION

In this paper, three new linear analog network-coding schemes have been proposed for two-way relay OFDM systems. These linear coding schemes are implemented with unitary matrices, hence the overall capacity will maintain the same as the conventional analog two-way relay OFDM systems. The main advantage of the proposed scheme lies in the capability of exploiting the frequency diversity made available in the product channels, hence SNR gains are obtained over the conventional analog two-way relay OFDM systems. Among the three proposed schemes, joint linear precoding at the source nodes and subcarrier permutation at the relay node is the most promising. Future extension of the current work includes precoding matrix optimization and performance analysis in the various scenarios.

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