Adaptive Tracking Control of Uncertain Robotic Manipulators in a Constrained Task Space

Keng Peng Tee and Haizhou Li

***Abstract*— In this paper, we present adaptive tracking control of uncertain robotic manipulators that operate in a constrained region of the task space. An asymmetric Barrier Lyapunov Function (BLF) is employed to ensure constraint satisfaction in the control design. By allowing the asymmetric barrier limits to vary with the desired trajectory in time, rather than fixing the barrier limit according to a worst case constant bound of the desired trajectory over time, we enlarge the set of feasible initial positions. Despite the derogatory transient effects of online parameter adaptation, asymptotic tracking of a desired trajectory is achieved without the end-effector ever transgressing the constrained region. The performance of the proposed adaptive control is illustrated through simulation.**

I. INTRODUCTION

Research in adaptive control of robotic manipulators has advanced considerably in recent decades to reduce dependency on a precise knowledge of the dynamics of the robot and the environment. Several important early works in adaptive control of robotic manipulators have established global convergent results [1], [2], [3], [4], [5]. While many works focused on joint space, robot motion control in task space is typically more intuitive and practically relevant. Adaptive control of robotic manipulators in task space has been proposed in [2], [6], [7]. The approach of [2] required computing an inverse of the Jacobian, which may encounter difficulties due to kinematic singularities. In [6], task-space composite adaptive control that avoids computing the inverse has been proposed. When dealing with non-redundant manipulators, the problem is tackled by directly inverting the Jacobian (see e.g. [7]). Besides traditional adaptive control that deals with unknown parameters in the robot manipulator, adaptive neural network control that deals with unknown functions has also been studied extensively [8], [9].

While adaptive control of robot manipulators has received much attention, the presence of hard position constraints in the robot's operating environment is usually neglected in the control design. The constraints may be imposed by physical obstacles in the surroundings that need to be avoided, or safety and performance limits that need to be respected at all times during robot operation. Violation of the constraints may result in performance degradation, hazards or system damage. As such, there is a need for rigorous handling of constraints in adaptive control of robots.

Barrier Lyapunov Functions (BLF) have been proposed to guarantee constraint satisfaction in control design. Unlike traditional Lyapunov functions (e.g. quadratic ones), which are radially unbounded, a BLF grows to infinity whenever its arguments approaches some limits. To guarantee that the barriers are not transgressed, it suffices to keep the BLF bounded in the closed loop system. To this end, BLF-based control design has been proposed for nonlinear systems in Brunovsky form [10] and strict feedback form [11], as well as for a class of electrostatic parallel plate microactuators [12].

This paper considers an uncertain two-degrees-of-freedom planar robotic manipulator whose end-effector needs to track a desired trajectory while always remaining inside a constrained region in the task space. We employ an asymmetric BLF, similar to that in [11], to ensure constraint satisfaction. However, our approach in this paper is novel in that it allows the asymmetric barrier limits to vary with the desired trajectory in time, rather than fixing the barrier limit according to a worst case constant bound of the desired trajectory over time. With both asymmetric and time-varying barrier limits, the set of feasible initial positions is significantly enlarged.

The remainder of this paper is organized as follows. In Section II, we present a model of a rigid two-degrees-of-freedom planar robotic manipulator, and briefly explain the idea of using a BLF for constraint satisfaction. Following that, in Section III, we present BLF-based adaptive backstepping control to ensure that the robot end-effector is constrained in a region of the task space. The simulation study in Section IV illustrates the performance of the control, and Section V presents concluding remarks.

II. PROBLEM FORMULATION AND PRELIMINARIES

Consider a rigid two-degree-of-freedom planar robotic manipulator described by:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) + F(\dot{q}) \quad = \quad \tau \tag{1}$$

where $M(q) \in \mathbb{R}^{2\times 2}$ is a symmetric positive definite matrix, $C(q,\dot{q})\dot{q} \in \mathbb{R}^2$ the Coriolis and centrifugal forces, $G(q) \in \mathbb{R}^2$ the gravitational forces, $F(\dot{q}) \in \mathbb{R}^2$ the frictional forces, $q \in \mathbb{R}^2$ the robot joint position, and $\tau \in \mathbb{R}^2$ the input torque. The terms $M(q)$, $C(q,\dot{q})$, and $G(q)$ contain uncertain parameters.

To track a desired trajectory in task space, the joint space dynamics (1) are transformed into task space dynamics [13]:

$$M_x(x)\ddot{x} + C_x(x,\dot{x})\dot{x} + G_x(x) + F_x(\dot{x}) \quad = \quad f \tag{2}$$

via the forward kinematics and the Jacobian:

$$x \quad = \quad \Omega(q), \quad \dot{x} = \frac{\partial \Omega}{\partial q}\dot{q} =: J(q)\dot{q} \tag{3}$$

K. P. Tee and H. Li are with the Institute for Infocomm Research, A*STAR, Singapore 138632 `kptee@i2r.a-star.edu.sg`, `hli@i2r.a-star.edu.sg`

where $x = [x_1, x_2]^T$ is the robot end-effector position, and the coefficient matrices are defined as

$$\begin{aligned} M_x &= J^{-T} M J^{-1}, \quad G_x = J^{-T} G, \quad F_x = J^{-T} F \\ C_x &= J^{-T}(C - M J^{-1} \dot{J}) J^{-1}, \quad f = J^{-T} \tau \end{aligned} \tag{4}$$

The following properties hold [13].

Property 1: The inertia matrix M_x is symmetric positive definite.

Property 2: The matrix $\dot{M}_x - 2C_x$ is skew symmetric.

Property 3: The left-hand-side expression of (2) can be linearly parameterized in terms of the robot system parameters as follows:

$$M_x(x)\phi_1 + C_x(x, \dot{x})\phi_2 + G_x(x) + F_x(\dot{x}) = \psi(\phi_1, \phi_2, x, \dot{x})\theta \tag{5}$$

for any $\phi_1, \phi_2 \in \mathbb{R}^2$, where $\theta \in \mathbb{R}^l$ are constant parameters and $\psi \in \mathbb{R}^l$ is a known regressor function.

For ease of control design, denote $\eta_1 = x$, $\eta_2 = \dot{x}$, and rewrite (2) into the following form suitable for backstepping:

$$\begin{aligned} \dot{\eta}_1 &= \eta_2 \\ \dot{\eta}_2 &= M_x^{-1}(\eta_1)(-C_x(\eta_1, \eta_2)\eta_2 - G_x(\eta_1) - F_x(\eta_2) + f) \end{aligned} \tag{6}$$

The control objective is to ensure that the end-effector position $x = [x_1, x_2]^T$ tracks a desired trajectory $x_d = [x_{d1}, x_{d2}]^T$ in the task space while keeping all closed loop signals bounded and preventing the position constraints $|x_i(t)| < k_{c_i}(t)$, $i = 1, 2$, from being violated.

Assumption 1: The constrained region Ω_x does not contain kinematic singularities. As a result, J is nonsingular as long as $x(t) \in \Omega_x \ \forall t \geq 0$.

Assumption 2: The desired trajectory $x_d(t)$ satisfies $x_d(t) \in \Omega_x \ \forall \ t \geq 0$, and there exist positive constants Y_1, Y_2, W_1, W_2 such that the time derivatives of $x_d(t)$ satisfy $|\dot{x}_{di}(t)| < Y_i, |\ddot{x}_{di}(t)| < W_i$, $i = 1, 2$, $\forall t \geq 0$.

Assumption 3: There exist positive constants C_i, D_i, E_i, $i = 1, 2$, such that the time-varying constraint $k_{c_i}(t)$ and its time derivatives satisfy $0 < k_{c_i}(t) \leq C_i$ and $|\dot{k}_{c_i}(t)| \leq D_i$, $|\ddot{k}_{c_i}(t)| \leq E_i$,$\forall t \geq 0$.

To prevent the robot end-effector from transgressing the constraints, we employ a Barrier Lyapunov Function [11]. The following lemma formalizes the result for general forms of barrier functions in Lyapunov synthesis satisfying $V_1(\xi) \to \infty$ as $|\xi| \to 1$, and is used to ensure that the time-varying output constraint is not violated.

Lemma 1: Let $\mathcal{Z} := \{\xi \in \mathbb{R}^m : |\xi_i| < 1, i = 1, ..., m\} \subset \mathbb{R}^m$ and $\mathcal{N} := \mathbb{R}^l \times \mathcal{Z} \subset \mathbb{R}^{l+m}$ be open sets. Consider the system

$$\dot{\eta} \ = \ h(t, \eta) \tag{7}$$

where $\eta := [w, \xi]^T \in \mathcal{N}$, and $h : \mathbb{R}_+ \times \mathcal{N} \to \mathbb{R}^{l+m}$ is piecewise continuous in t and locally Lipschitz in η, uniformly in t, on $\mathbb{R}_+ \times \mathcal{N}$. Let $\mathcal{Z}_i := \{\xi_i \in \mathbb{R} : |\xi_i| < 1\} \subset \mathbb{R}$. Suppose that there exist functions $U : \mathbb{R}^l \to \mathbb{R}_+$ and $V_i : \mathcal{Z}_i \to \mathbb{R}_+$, $i = 1, ..., m$, continuously differentiable and positive definite in their respective domains, such that

$$V_i(\xi_i) \to \infty \ \text{ as } \ |\xi_i| \to 1, \quad i = 1, ..., m \tag{8}$$

$$\gamma_1(\|w\|) \leq U(w) \leq \gamma_2(\|w\|) \tag{9}$$

where γ_1 and γ_2 are class K_∞ functions. Let $V(\eta) := \sum_{i=1}^m V_i(\xi_i) + U(w)$, and $\xi(0) \in \mathcal{Z}$. If the inequality holds:

$$\dot{V} \ = \ \frac{\partial V}{\partial \eta} h \ \leq \ 0 \tag{10}$$

in the set $\xi \in \mathcal{Z}$, then $\xi(t) \in \mathcal{Z} \ \forall t \in [0, \infty)$.

Proof: The proof is similar to that of [11, Lemma 1], and is provided here for completeness.

The conditions on h ensure the existence and uniqueness of a maximal solution $\eta(t)$ on the time interval $[0, \tau_{\max})$, according to [14, p.476 Theorem 54]. From the fact that $\xi(0) \in \mathcal{Z}$, we know that $V_i(\xi_i(0))$, $i = 1, ..., n$, and thus $V(\eta(0))$, exist.

Since $V(\eta)$ is positive definite and $\dot{V} \leq 0$ in the set $\xi \in \mathcal{Z}$, it follows that $V(\eta(t)) \leq V(\eta(0)) \ \forall t \in [0, \tau_{\max})$. From $V(\eta) = \sum_{i=1}^n V_i(\xi_i) + U(w)$ and the fact that $V_i(\xi_i)$, $i = 1, ..., n$ are positive functions, it is clear that each $V_i(\xi_i(t))$ is bounded $\forall t \in [0, \tau_{\max})$. Since $V_i(\xi_i) \to \infty$ only if $\xi_i \to \pm 1$, we conclude, from the boundedness of $V_i(\xi_i(t))$, that $|\xi_i(t)| < 1 \ \forall t \in [0, \tau_{\max})$.

Therefore, there is a compact subset $K \subseteq \mathcal{N}$ such that the maximal solution of (7) satisfies $\eta(t) \in K \ \forall t \in [0, \tau_{\max})$. As a direct consequence of [14, p.481 Proposition C.3.6], we have that $\eta(t)$ is defined $\forall t \in [0, \infty)$. It follows that $\xi(t) \in \mathcal{Z} \ \forall t \in [0, \infty)$. ■

III. ADAPTIVE CONTROL DESIGN IN CONSTRAINED TASK SPACE

The design is based on adaptive backstepping [15], detailed as follows.

Step 1 Denote $z = [z_1, z_2]^T = \eta_1 - x_d$ and $v = [v_1, v_2]^T = \eta_2 - \alpha$, where $\alpha = [\alpha_1, \alpha_2]^T$ is a stabilizing function to be designed shortly. Consider the asymmetric barrier function:

$$\begin{aligned} V_1 \ = \ \frac{1}{2} \sum_{i=1}^{2} \Bigg(& p(z_i) \log \frac{k_{b_i}^2(t)}{k_{b_i}^2(t) - z_i^2} \\ & + (1 - p(z_i)) \log \frac{k_{a_i}^2(t)}{k_{a_i}^2(t) - z_i^2} \Bigg) \end{aligned} \tag{11}$$

where

$$k_{a_i}(t) \ := \ k_{c_i}(t) + x_{di}(t) \tag{12}$$

$$k_{b_i}(t) \ := \ k_{c_i}(t) - x_{di}(t) \tag{13}$$

$$p(\bullet) \ := \ \begin{cases} 1, & \text{if } \bullet > 0 \\ 0, & \text{if } \bullet \leq 0 \end{cases} \tag{14}$$

for $i = 1, 2$. Due to Assumptions 2-3, there exist positive constants $\underline{k}_{b_1}$, $\overline{k}_{b_1}$, $\underline{k}_{a_1}$ and $\overline{k}_{a_1}$ such that

$$\begin{aligned} 0 < \underline{k}_{b_1} \leq \ & k_{b_1}(t) \ \leq \overline{k}_{b_1}, \quad \forall \ t \geq 0 \\ 0 < \underline{k}_{a_1} \leq \ & k_{a_1}(t) \ \leq \overline{k}_{a_1}, \quad \forall \ t \geq 0 \end{aligned} \tag{15}$$

By a change of coordinates

$$\xi_{a_i} = \frac{z_i}{k_{a_i}}, \quad \xi_{b_i} = \frac{z_i}{k_{b_i}} \tag{16}$$

$$\xi_i = \begin{cases} \xi_{a_i}, & z_i \leq 0 \\ \xi_{b_i}, & z_i > 0 \end{cases} \tag{17}$$

for $i = 1, 2$, we can rewrite (11) as

$$V_1 = \frac{1}{2}\sum_{i=1}^{2} \log \frac{1}{1-\xi_i^2} \tag{18}$$

The time derivative of V_1 is given by

$$\begin{aligned} \dot{V}_1 = & \sum_{i=1}^{2}\left[\frac{p(z_i)\xi_{b_i}}{k_{b_i}(1-\xi_{b_i}^2)}\left(v_i + \alpha_i - \dot{x}_{d_i} - z_i\frac{\dot{k}_{b_i}}{k_{b_i}}\right)\right. \\ & \left. + \frac{(1-p(z_i))\xi_{a_i}}{k_{a_i}(1-\xi_{a_i}^2)}\left(v_i + \alpha_i - \dot{x}_{d_i} - z_i\frac{\dot{k}_{a_i}}{k_{a_i}}\right)\right] \end{aligned} \tag{19}$$

Design the stabilizing function α as:

$$\begin{aligned} \alpha_i = & -\left(\frac{p(z_i)}{k_{b_i}^2} + \frac{(1-p(z_i))}{k_{a_i}^2}\right)\kappa_{z_i} z_i^3 + \dot{x}_{d_i} \\ & - z_i\sqrt{\left(\frac{\dot{k}_{a_i}}{k_{a_i}}\right)^2 + \left(\frac{\dot{k}_{b_i}}{k_{b_i}}\right)^2 + \lambda}, \quad i = 1, 2 \end{aligned} \tag{20}$$

where λ and κ_{z_i} are positive constants. It can be shown that $\lim_{z_i \to 0^+} \partial\alpha_i/\partial z_i = \lim_{z_i \to 0^-} \partial\alpha_i/\partial z_i$, and thus, α_i, $i = 1, 2$, are continuously differentiable. The last term of (20) is designed to dominate the term $-z_i\dot{k}_{b_i}/k_{b_i}$ when $z_i > 0$, and the term $-z_i\dot{k}_{a_i}/k_{a_i}$ when $z_i \leq 0$ in (19), since

$$\sqrt{\left(\frac{\dot{k}_{a_i}}{k_{a_i}}\right)^2 + \left(\frac{\dot{k}_{b_i}}{k_{b_i}}\right)^2 + \lambda} + p\frac{\dot{k}_{b_i}}{k_{b_i}} + (1-p)\frac{\dot{k}_{a_i}}{k_{a_i}} \geq 0 \tag{21}$$

Then, the time derivative of V_1 satisfies

$$\begin{aligned} \dot{V}_1 \leq & -\sum_{i=1}^{2}\frac{\kappa_{z_i}\xi_i^4}{1-\xi_i^2} \\ & + \sum_{i=1}^{2}\left(\frac{p(z_i)\xi_{b_i}v_i}{k_{b_i}(1-\xi_{b_i}^2)} + \frac{(1-p(z_i))\xi_{a_i}v_i}{k_{a_i}(1-\xi_{a_i}^2)}\right) \end{aligned} \tag{22}$$

From (16)-(17) and the fact that $k_{a_i}, k_{b_i} > 0$, we note that $\xi_i > 0$ when $z_i > 0$ and that $\xi_i \leq 0$ when $z_i \leq 0$. Hence, the first term on the right hand side of (22) is nonpositive in the set $\xi = [\xi_1, \xi_2]^T \in \mathcal{Z}$, where

$$\mathcal{Z} := \{(\xi_1, \xi_2) \in \mathbb{R}^2 : |\xi_i| < 1, \ i = 1, 2\} \tag{23}$$

The second term is eliminated in the second step.

Step 2 The control τ will be designed in this step. Consider the Lyapunov function candidate:

$$V_2 = V_1 + \frac{1}{2}v^T M_x(\eta_1)v + \frac{1}{2}\tilde{\theta}^T\Gamma^{-1}\tilde{\theta} \tag{24}$$

where $\tilde{\theta} := \hat{\theta} - \theta$ and $M_x(\eta_1)$ is positive definite from Property 1. The time derivative of V_2 is given by

$$\begin{aligned} \dot{V}_2 = & \ v^T[-C_x(\eta_1,\eta_2)(v+\alpha) - G_x(\eta_1) - F_x(\eta_2) + f \\ & -M_x(\eta_1)\dot{\alpha}] + \frac{1}{2}v^T\dot{M}_x v + \tilde{\theta}^T\Gamma^{-1}\dot{\hat{\theta}} + \dot{V}_1 \\ = & \ v^T[-C_x(\eta_1,\eta_2)\alpha - G_x(\eta_1) - F_x(\eta_2) + f \\ & -M_x(\eta_1)\dot{\alpha}] + \frac{1}{2}v^T[\dot{M}_x(\eta_1,\eta_2) - 2C_x(\eta_1,\eta_2)]v \\ & + \tilde{\theta}^T\Gamma^{-1}\dot{\hat{\theta}} + \dot{V}_1 \end{aligned} \tag{25}$$

where

$$\dot{\alpha} = \frac{\partial\alpha}{\partial\eta_1}\eta_2 + \sum_{j=0}^{1}\frac{\partial\alpha}{\partial x_d^{(j)}}x_d^{(j+1)} + \frac{\partial\alpha}{\partial t} \tag{26}$$

Due to Property 2, $v^T(\dot{M}_x - 2C_x)v = 0$. Furthermore, with Property 3, we can rewrite (25) in the linearly parameterized form:

$$\dot{V}_2 = v^T[-\psi(\dot{\alpha}, \alpha, \eta_1, \eta_2)\theta + f] + \tilde{\theta}^T\Gamma^{-1}\dot{\hat{\theta}} + \dot{V}_1 \tag{27}$$

Design the adaptive control law as

$$\dot{\hat{\theta}} = \Gamma\psi^T v \tag{28}$$

$$f = -K_v v + \psi\hat{\theta} + g \tag{29}$$

$$\tau = J^T f \tag{30}$$

where $K_v := \text{diag}(\kappa_{v_1}, \kappa_{v_2}) > 0$, and $g = [g_1, g_2]^T$, with

$$g_i := -\left(\frac{p(z_i)\xi_{b_i}}{k_{b_i}(1-\xi_{b_i}^2)} + \frac{(1-p(z_i))\xi_{a_i}}{k_{a_i}(1-\xi_{a_i}^2)}\right) \tag{31}$$

for $i = 1, 2$, is used to cancel the coupling term in (22). Thus, it can be shown that

$$\begin{aligned} \dot{V}_2 \leq & -\sum_{i=1}^{2}\left(\frac{p(z_i)\kappa_{z_i}\xi_{b_i}^4}{1-\xi_{b_i}^2} + \frac{(1-p(z_i))\kappa_{z_i}\xi_{a_i}^4}{1-\xi_{a_i}^2}\right) \\ & - z_2^T K_2 z_2 \end{aligned} \tag{32}$$

which is nonpositive in the set $\xi \in \mathcal{Z}$, where $\mathcal{Z}$ is defined in (23).

Theorem 1: Consider the planar robot (1) under the adaptive controller (20), (28)-(30), and Assumptions 1-2. If the initial conditions are such that

$$|x_i(0)| < k_{c_i}(0), \quad i = 1, 2 \tag{33}$$

then the following properties hold.

i) The tracking error $z = [z_1, z_2]^T$ satisfies

$$-k_{a_i}(t) < z_i(t) < k_{b_i}(t), \quad i = 1, 2, \quad \forall\ t > 0 \tag{34}$$

ii) The end-effector position $x(t)$ satisfies $|x_i(t)| < k_{c_i}(t)$, $i = 1, 2$, $\forall t > 0$, i.e. the constraint is never violated.

iii) All closed loop signals are bounded.

iv) The tracking error $z(t)$ converges to zero asymptotically, i.e., $x(t) \to x_d(t)$ as $t \to \infty$.

Proof:

i) The closed loop system can be written as

$$\begin{aligned}\dot{z} &= v - \left(\frac{p(z_i)}{k_{b_i}^2} + \frac{(1-p(z_i))}{k_{a_i}^2}\right)\kappa_{z_i} z_i^3 \\ &\quad - z_i\sqrt{\left(\frac{\dot{k}_{a_i}}{k_{a_i}}\right)^2 + \left(\frac{\dot{k}_{b_i}}{k_{b_i}}\right)^2 + \lambda} \\ \dot{v} &= M_x^{-1}(\psi\tilde{\theta} - K_v v + g) \\ \dot{\tilde{\theta}} &= \Gamma\psi^T v \end{aligned} \tag{35}$$

where the right hand side is piecewise continuous in t and locally Lipschitz in $(z, v, \tilde{\theta})$, uniformly in t. From (32), we know that $\dot{V}_2 \leq 0$ in the set $\xi \in \mathcal{Z}$. Then, with the initial conditions (33) and the help of Lemma 1, we have $|\xi_i(t)| < 1$, $i = 1, 2$, $\forall\ t > 0$.

From (17), consider $z_i(t) \leq 0$ for some $t > 0$, which yields $-1 < \xi_{a_i}(t) \leq 0$. Since $\xi_{a_i} = z_i / k_{a_i}$ for $z_i \leq 0$, and $k_{a_i} > 0$, we obtain

$$-k_{a_i}(t) < z_i(t) \leq 0 \tag{36}$$

for $i = 1, 2$. Similarly, considering $z_i(t) > 0$ for some $t > 0$ yields $0 < \xi_{b_i}(t) \leq 1$ and, in turn,

$$0 < z_i(t) < k_{b_i}(t) \tag{37}$$

for $i = 1, 2$. Combining both cases, we conclude that

$$-k_{a_i}(t) < z_i(t) < k_{b_i}(t), \quad i = 1, 2, \quad \forall\ t > 0$$

ii) Since $x_i(t) = z_i(t) + x_{di}(t)$ and $-k_{a_i} < z_i(t) < k_{b_i}$, for $i = 1, 2$, we infer that

$$-k_{a_i}(t) + x_{di}(t) < x_i(t) < k_{b_i}(t) + x_{di}(t) \tag{38}$$

for all $t > 0$. From the definitions of k_{a_i} and k_{b_i} in (12) and (13) respectively, we conclude that $|x_i(t)| < k_{c_i}(t)$, $i = 1, 2$, $\forall t > 0$.

iii) Since $|\xi_i(t)| < 1$, $i = 1, 2$, $\forall t > 0$, it is clear that $V_2(t) \leq V_2(0)$, and hence $\hat{\theta}(t)$ and $z_2(t)$ are bounded $\forall\ t > 0$. From (20), we know that the stabilizing function $\alpha_1(t)$ is bounded, since $|\dot{x}_{di}(t)| < Y_i$, $-\underline{k}_{a_i} < z_i(t) < \overline{k}_{b_i}$, $|\dot{k}_{b_i}| = |\dot{k}_{a_i}| \leq D_i + Y_i$, and k_{a_i}, k_{b_i} are bounded away from 0. This leads to the boundedness of $x_2(t)$, since $x_2 = z_2 + \alpha_1$. Since f is a continuous function of bounded signals in the set $|\xi_i| < 1$, $i = 1, 2$, we know that $f(t)$ is bounded $\forall\ t > 0$. From Assumption 1, the Jacobian J is smooth and nonsingular, and thus, $\tau(t)$ is bounded $\forall\ t > 0$. Hence, all closed loop signals are bounded.

iv) Let

$$\rho := \sum_{i=1}^{2}\left(\frac{p(z_i)\kappa_{z_i}\xi_{b_i}^4}{1-\xi_{b_i}^2} + \frac{(1-p(z_i))\kappa_{z_i}\xi_{a_i}^4}{1-\xi_{a_i}^2}\right)$$

Since $|\xi_i(t)| < 1$, $i = 1, 2$, $\forall t > 0$, it can be shown that $\lim_{t\to\infty}\int_0^t \rho(\tau)\, d\tau < \infty$ and that $\dot{\rho}(t)$ is bounded. Then, Barbalat's Lemma [16] is used to show that $\rho(t) \to 0$ as $t \to \infty$. Thus, $\xi_i(t) \to 0$ as $t \to \infty$. Since $k_{a_i}(t), k_{b_i}(t)$ are bounded away from 0 $\forall\ t > 0$, we have $\xi_i = 0 \Rightarrow z_i = 0$, and conclude that $z_i(t) \to 0$ as $t \to \infty$, $i = 1, 2$. ■

IV. SIMULATION

In the simulation study, we consider a two-link frictionless robot moving in a horizontal plane subject to rectangular and elliptic regions of constraint in the task space. The robot dynamics are modeled by the following equations [1]

$$\begin{aligned}\tau_1 &= m_2 l_2^2(\ddot{q}_1 + \ddot{q}_2) + m_2 l_1 l_2 c_2(2\ddot{q}_1 + \ddot{q}_2) \\ &\quad + (m_1 + m_2) l_1^2 \ddot{q}_1 - m_2 l_1 l_2 s_2 \dot{q}_2^2 - 2 m_2 l_1 l_2 s_2 \dot{q}_1 \dot{q}_2 \\ \tau_2 &= m_2 l_2^2(\ddot{q}_1 + \ddot{q}_2) + m_2 l_1 l_2 c_2 \ddot{q}_1 + m_2 l_1 l_2 s_2 \dot{q}_1^2 \end{aligned} \tag{39}$$

where $c_i = \cos(q_i)$ and $s_i = \sin(q_i)$. The uncertain parameters are m_1 and m_2, whose true values are $1.5kg$ and $1.0kg$ respectively. The lengths of the links are $l_1 = l_2 = 0.3m$.

The design parameters are selected as $\lambda = 1.0$, $\Gamma = 1.0I$, $K_v = 1.0I$, $K_z = 4.0I$. Initially, the end-effector is at rest at the origin. The desired trajectory traces a circular path in the task space, and is described by

$$\begin{aligned} x_{d1}(t) &= 0.14\cos(0.5t) \\ x_{d2}(t) &= 0.14\sin(0.5t) \end{aligned} \tag{40}$$

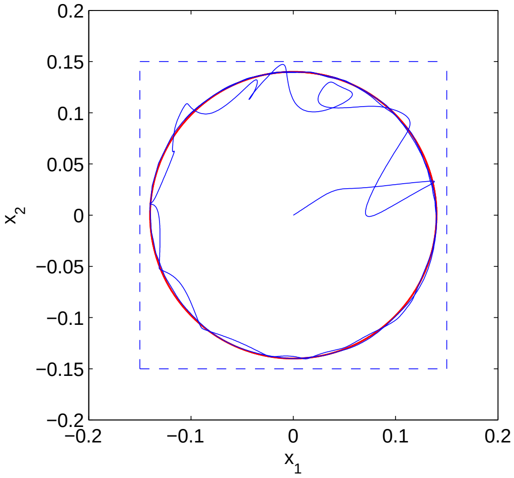

Fig. 1. The robot end-effector (solid blue line), starting from the origin, tracks the circular desired trajectory (red line) while always remaining in the static constrained region.

A. Static Constraints

The problem of static constraints can be tackled by extending the approach of [12] to two dimensions, defining a barrier limit in such a way that it depends on a worst case constant bound of the desired trajectory over time. In this paper, we take a different approach by allowing the barrier limit to vary with the desired trajectory in time, thus making the design significantly less conservative.

We consider a rectangular constraint region, where $x_1(t)$ and $x_2(t)$ are to satisfy

$$|x_i(t)| < k_{c_i} = 0.15, \quad i = 1, 2, \quad \forall\ t \geq 0$$

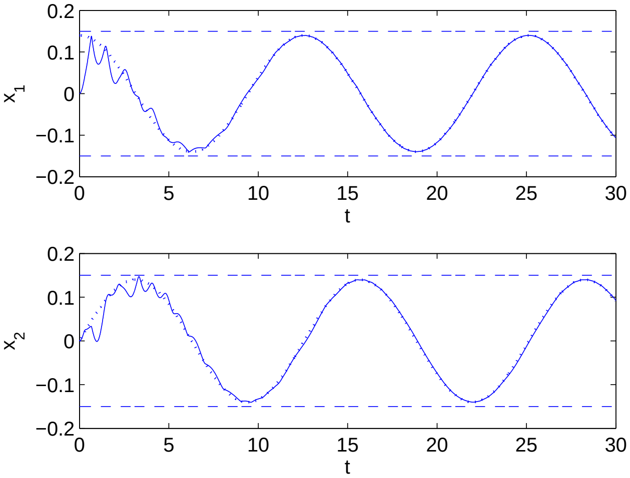

Fig. 2. Robot end-effector trajectories under static constraints.

where k_{c_1} and k_{c_2} are positive constants.

Figure 1 shows that despite the desired trajectory approaching close to the boundaries of the constrained region, and despite the transients of online parameter estimation, the robot end-effector never transgresses the position constraints in task space. The actual trajectories of x_1 and x_2 in time are shown in Figure 2, where it it clear that $|x_1(t)| < 0.15$ and $|x_2(t)| < 0.15 \ \forall\ t > 0$.

Figure 3 shows that the tracking errors $z_i \ i = 1, 2$, converge to zero and never transgresses the asymmetric and time-varying barriers, i.e. $-k_{a_i}(t) < z_i(t) < k_{b_i}(t) \ \forall\ t > 0$.

Throughout the process, the parameter estimates $\hat{\theta}_1$ and $\hat{\theta}_2$, and well as the input torque τ, are bounded, as shown in Figure 4. The parameter estimates converge to values that are different from the true values of the parameters. This suffices for our purpose of trajectory trajectory, and is not surprising since persistency of excitation conditions are not invoked. The input torques exhibit intermittent peaks. This is due to $z_i(t)$ approaching $k_{b_i}(t)$ or $-k_{a_i}(t)$ for some t, which means $\xi_a(t) \to -1$ or $\xi_b(t) \to 1$, and thus leading to the growth of the term g in the control, as described in (31).

B. Dynamic Constraints

When considering dynamic constraints, k_{c_i}, $i = 1, 2$, are no longer constant, but time-varying. In the constraint region, the position vector x is required to satisfy

$$|x_i(t)| < k_{c_i}(t) = a_i\sqrt{1 - \frac{x_{dj}^2(t)}{a_j^2}}, \quad \forall\ t \geq 0 \tag{41}$$

where $i = 1, 2$, $j = 1, 2$, $j \neq i$, $a_1 = 0.2$ and $a_2 = 0.15$.

Simulated performance is similar to that for rectangular constraints, and some of the results are shown in Figures 5-6. The robot end-effector converges to the desired trajectory while staying within the constrained region, withstanding any perturbing effects from online parameter adaptation. Due to the difference in the shape of the constrained region, the profile of the barriers in Figure 6 are distinct from that in Figure 3 for rectangular constraints.

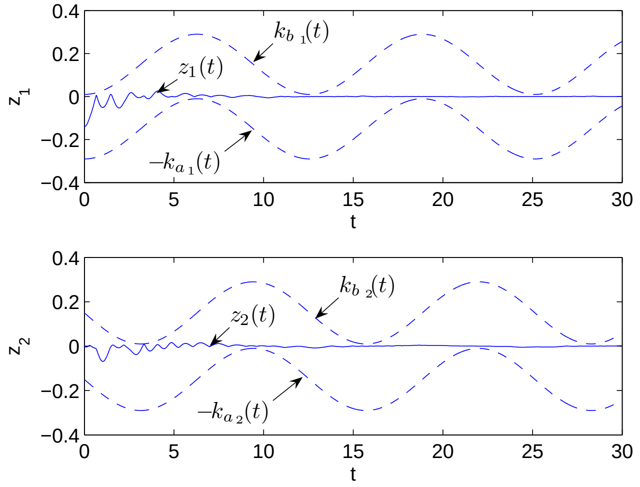

Fig. 3. The tracking errors for the case of static constraints converge to zero and never transgresses the time-varying barriers.

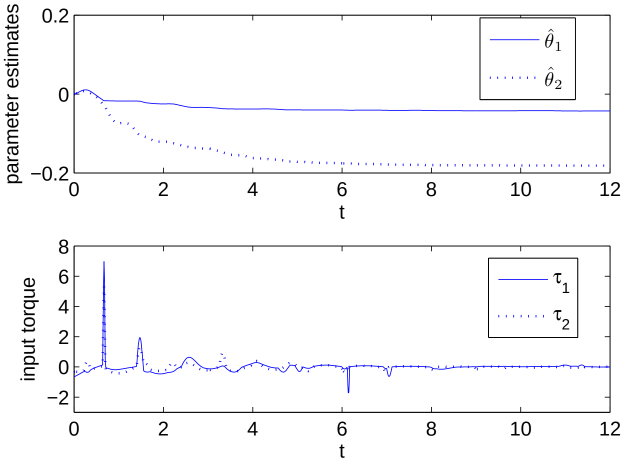

Fig. 4. The parameter estimates and input torque are bounded.

V. CONCLUSIONS

We have presented adaptive tracking control of uncertain robotic manipulators that operate in a constrained region of the task space. In particular, we considered a planar robot operating within rectangular and elliptic regions of constraint. An asymmetric Barrier Lyapunov Function (BLF) has been employed in the control design to guarantee that the constraints are not transgressed. We have shown that asymptotic tracking of a desired trajectory is achieved without the end-effector transgressing the constrained region, and that all closed loop signals are bounded. By incorporating both asymmetric and time-varying barrier limits in the current work, we have enlarged the set of feasible initial positions significantly. The performance of the proposed control has

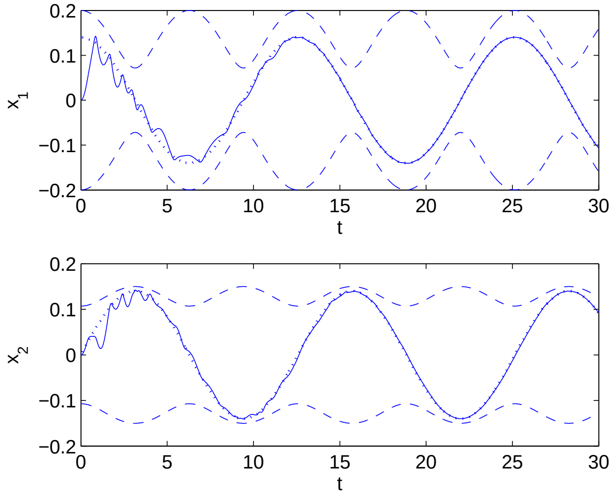

Fig. 5. Robot end-effector trajectories in the presence of dynamic constraints.

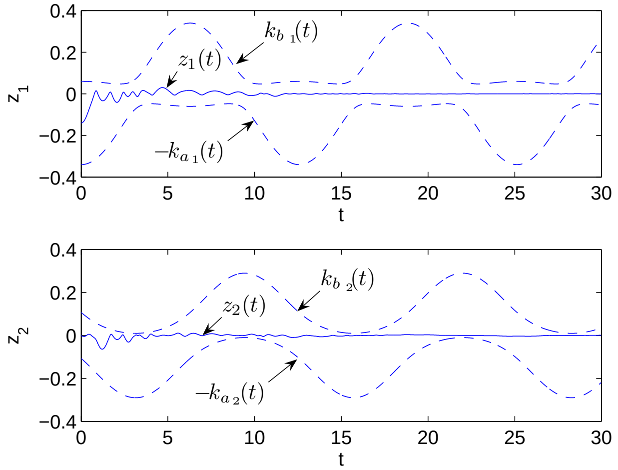

Fig. 6. The tracking errors in the presence of dynamic constraints.

been illustrated through simulation of the robot manipulator motion under static and dynamic constraints.

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