Edge based Wideband Sensing for Cognitive Radio: Algorithm and Performance Evaluation

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Abstract—Since a cognitive radio does not have fixed spectra, it may need to sense a very large frequency range to find an available band. The sensed aggregate bandwidth could be as large as several GHz. This is especially challenging if the center frequencies and bandwidths of the sensed signals are unknown and need to be detected. In this paper, an edge based wideband sensing is proposed. The method first uses the product of wavelet transforms at different scales to detect the edges (sharp changing points) of the power spectral density (PSD) of the received signal. It then forms the possible bands based on the detected edges. Thereafter, it applies a multi-band detection scheme to classify the bands as occupied or vacant. Finally, the signal to noise ratio (SNR) of each occupied band is estimated. Performance evaluation is also a complicated issue for wideband sensing. Other than the conventional metrics as probability of detection and probability of false alarm, three new criteria are proposed to evaluate the performance of a wideband sensing. Simulations are provided to verify the methods.

Keywords: Cognitive radio, spectrum sensing, wideband sensing, multi-band sensing, subcarrier sensing, signal detection, edge detection, wavelet

I. INTRODUCTION

In recent years, cognitive radio has become a hot topic in academic research as well as in standards and government regulations [1], [2]. The basic idea of cognitive radio is to allow different networks/users to dynamically sharing the spectrum to achieve maximum utility without causing harmful interference to each other. To do so, a cognitive radio needs to frequently perform spectrum sensing, i.e., detecting the presence of other primary/secondary users [3], [4], [5], [6], [7]. Based on the detected spectrum information, a cognitive radio takes actions to maximize its own cost function and at the same time protect the interests of primary and other secondary users.

As a cognitive radio does not have fixed spectra, it may need to sense a very large frequency range to find an available band for transmission. The sensed aggregate spectral bandwidth could be as large as several GHz. Furthermore, a cognitive radio may not have enough information about the band policy of primary users and other cognitive radios. Here band policy means some rules on the spectrum usage that users must obey. For example, the allocated center frequency and bandwidth for a TV broadcaster are usually within a pre-defined set (called bands or channels). When we sense TV signals in a large frequency range, we can sense band by band, which is called multi-band sensing. However, in some other situations, the sensed primary/secondary users may not have such policy or we do not know the policy. In other words, the sensed signal can be at any center frequency and occupy any bandwidth within the large frequency range. Lack of information of the center frequency and bandwidth of the sensed signal means that the band by band or channel by channel sensing is not valid anymore. The center frequency and bandwidth of the sensed signal also need to be detected. We call this wideband sensing, which is more general than the conventional multi-band sensing where the center frequency and bandwidth of each band are known. Hence multi-band sensing is just a special case of wideband sensing. Please note that the two terms may not be differentiated properly in some literature.

A fundamental challenging for wideband sensing is the requirement of very high sampling rate. Fortunately there have been solutions to ease this requirement based on the recently proposed compressed sampling [8], [9], [10], [11]. For conventional multi-band sensing, there already have been some researches [12], [13], [14], [15]. However, for general wideband sensing, research is still limited. One known method is based on the wavelet edge detection [16], [17], [18]. The method assumes that the power spectral density (PSD) of the received wideband signal has sharp changes at the two ends of each occupied band. The ends of each occupied band are therefore similar to edges in an image or steps in a curve. The edge or step detection method can be readily used to detect such ends [19], [20]. However, the real situation is much more complicated: (1) the signal may also have sharp changes within a band; (2) at low signal to noise ratio (SNR) level, the edges contributed by noise may have significant impact; (3) the ends of an occupied band may not be sharp enough. In this paper, we will study these problems and improve the algorithm. Major contributions of the proposed algorithm are as follows. (1) The wavelet product is normalized by the mean of the PSD to combat noise uncertainty; (2) A threshold is proposed for the local maxima of the normalized wavelet product to prevent keeping too many fake edges; (3) After forming the possible bands based on the detected edges, a multi-band detection is proposed to classify the bands as occupied or vacant; (4) A method is proposed to estimate the SNR of each occupied band.

Not only the algorithm, performance evaluation is also much more complicated in wideband sensing than that in single-band or multi-band sensing. For single/multi-band sensing, we can use the probability of detection and the probability of false alarm to evaluate the performance of a detection method. However, since the center frequencies and bandwidths of any possible bands are unknown in wideband sensing, the accuracy
of finding the center frequencies and bandwidths becomes a major performance indication. We need to define adequate metrics for evaluating this performance as well. Thus, except for probability of detection and the probability of false alarm, we propose three new criteria to evaluate the wideband sensing performance: subcarrier occupancy error rate (SOER), band occupancy error (BOE) and wideband spectral error (WSE). The three criteria evaluate a given method from different aspects.

The rest of the paper is organized as follows. The system model is given in Section II. The edge based wideband sensing is discussed in Section III, where multi-scale wavelet is used for the edge detection. A multi-band sensing is discussed in Section IV. In Section V, performance metrics are introduced to evaluate wideband sensing methods. Simulation results are given in Section VI. Finally conclusions and future research direction are discussed in Section VII.

II. SYSTEM DESCRIPTION

To find a good available frequency band, a cognitive radio is better to sense a frequency range as large as possible. Assume that the cognitive radio chooses to sense the frequency range with center frequency \( f_c \) and bandwidth \( B_w \). The bandwidth \( B_w \) could be as large as several GHz. Within the frequency range, it is assumed that the primary or other secondary user’s signal could be at any place and occupy any amount of bandwidth.

Let \( \mathcal{H}_0 \) and \( \mathcal{H}_1 \) represent the hypothesis of signal absent and signal present in the whole frequency range, respectively. Then the received signal (after sampling) is given by

\[
\begin{align*}
\mathcal{H}_0: & \quad x(n) = \eta(n), \\
\mathcal{H}_1: & \quad x(n) = s(n) + \eta(n),
\end{align*}
\]

where \( N_s \) is the number of samples, \( \eta(n) \) is the noise, and \( s(n) \) is the received signal component (received source signal) at time \( n \), which includes multipath and fading effects. A wideband sensing usually has three objectives. (1) The first objective, similar to conventional single/multi-band sensing, is to make a decision on the binary hypothesis testing (choose \( \mathcal{H}_0 \) or \( \mathcal{H}_1 \)) based on the received signal. For this objective, the performance is generally indicated by two probabilities: probability of detection, \( P_d \), which defines, at the hypothesis \( \mathcal{H}_1 \), the probability of the sensing algorithm having detected the presence of the source signal; probability of false alarm, \( P_f \), which defines, at the hypothesis \( \mathcal{H}_0 \), the probability of the sensing algorithm claiming the presence of the source signal. (2) The second objective is to estimate the exact spectrum locations of the source signal within the frequency range, if the decision is \( \mathcal{H}_1 \). That is, we need to properly divide the whole frequency range into possibly a number of smaller bands and classify each band as occupied or vacant. The performance metric for this objective is much more complicated and will be discussed later. (3) Finally, the third objective is to estimate the average SNR of each occupied band detected. This is to provide information for possible spectrum sharing with the existing users in the band. It is possible that further information on each band, such as signal features, is required to be detected.

To sense a very large frequency range, typically we need a very large sampling rate, which is very challenging in practical implementation. Fortunately, assuming that a large part of the frequency range is vacant, that is, the signal is frequency domain sparse, we can use the recently developed compressed sampling (also called compressed sensing) to reduce the sampling rate by a large margin \([8], [9], [10], [11] \). At highly reduced sampling rate, the received signal can be recovered with very high probability \([8], [9], [10] \). This technique could be used for the proposed methods in this paper to reduce the sampling complexity. However, it does not affect the detection methods if we assume that the received signal is fully recovered without error from the compressed samples. Hence, in the following we use uncompressed samples in all cases, although compressed sampling may be used.

As usual, we assume that the received signal has been down-converted to the baseband. Hence the sensed frequency range is \([-B_w/2, B_w/2] \). Assume that the noise samples \( \eta(n) \) are independent and identically distributed (i.i.d) complex Gaussian. It is assumed that no information on the sensed source signals is available at the receiver. Therefore, \( s(n) \) is modeled as a random variable with unknown distribution. It is also assumed that signal and noise are independent.

III. EDGE BASED WIDEBAND SENSING

Usually, a signal should occupy one or multiple continuous frequency ranges. A continuous frequency range is usually called a band. Theoretically, the average PSD of an occupied band should be higher than the noise power, while the average PSD of a vacant band should be the noise power. Hence, at the intersection of an occupied band and a vacant band there is a sharp change \([16] \). The sharp changing point is called an edge or a step. There have been a few researches in edge or step detection \([19], [20] \). These can be readily used here \([16] \). However, the real situation in cognitive radio is much more complicated: (1) the source signal may also have sharp changes within an occupied band; (2) since a cognitive radio must have the capability to detect primary signals at very low SNR level to avoid the “hidden node problem”, the edges contributed by noise may have significant impact; (3) the ends of an occupied band may not be sharp enough. To overcome these difficulties, we need other solutions.

To compute the PSD, an easy and direct method is using the fast Fourier transform (FFT), though other methods could be used to improve the performance. Let \( N \) be the size of FFT, which means that the frequency range is sampled to have \( N \) frequency points. Each frequency point is called a subcarrier. Let \( X_i \), \( S_i \) and \( \Gamma_i \) be the normalized FFT of the received signal vector \( x(iN : (i+1)N-1) \), the source signal vector \( s(iN : (i+1)N-1) \) and noise vector \( \eta(iN : (i+1)N-1) \), respectively. Note that the Matlab notation is used here and in the following. Let \( P(n) = \eta(n) = 0, 1, \cdots , N-1 \) be the PSD at
subcarrier \(n\) and
\[
P = \begin{bmatrix} P(0) & P(1) & \cdots & P(N-1) \end{bmatrix}^T.
\]

Then an estimation of the PSD is given as
\[
P = \frac{1}{M} \sum_{i=0}^{M-1} \mathbf{X}_i \odot \mathbf{X}_i^*,
\]
where \(M = \lfloor N_e/N \rfloor\), \(\odot\) is the piecewise product and \(\ast\) is the conjugate. Note that the received signal \(x(n)\) is zero padded if necessary.

The multi-scale wavelet was used for step detection in [19], [20]. Here we briefly review the method. Let \(\varphi(t)\) be a continuous-time wavelet and \(W_a f(y)\) be the continuous wavelet transform of signal \(f(t)\), where \(a\) is the scale of the wavelet. When the input signal is a discrete sequence, its wavelet transform is the approximation of the integration in the continuous wavelet transform by a discrete convolution. We define \(W_a f(n)\) as the wavelet transform of discrete sequence \(f(m)\). It is suggested in [20], [16] that using the dyadic scale, that is, \(a = 2^j\), is preferable. Also it is shown that using the product of wavelet transforms at multi-scales gives better performance at low SNR [20]. Let \(f(n)\) be the product of the wavelet transforms of \(P(n)\) at the first \(J\) dyadic scales, that is,
\[
F(n) = \prod_{j=1}^{J} W_{2^j} P(n).
\]

It is known that, at the step or edge, \(|F(n)|\) will have a relatively larger value. Hence, the local maximum points of \(|F(n)|\) could give the locations of edges. However, there may be large number of local maximum points. Some of them do represent the band edges, but others may be caused by noise and sharp changes of signal within a band. How to choose the local maximum points is a challenging problem. The known research [16], [17] has little study on this.

To prevent keeping too many fake edges, we should set a threshold for the local maxima. We should only keep the local maximum points which are larger than the threshold. The choice of the threshold is a challenging task. Note that a local maximum of \(|F(\cdot)|\) at point \(n\) is not only related to the shape of PSD near \(n\) but also related to the amplitude of PSD. So the threshold should not be a fixed value. We propose to normalize the wavelet transform product first and then choose a fixed threshold. Let \(\Delta\) be the mean of the PSD, that is,
\[
\Delta = \frac{1}{N} \sum_{n=0}^{N-1} P(n).
\]

Let \(F_N(n)\) be the normalized product, that is,
\[
F_N(n) = |F(n)|/\Delta^J.
\]

Let \(\gamma_e\) be the threshold, which is fixed for a given FFT size \(N\) and sample size \(N_e\). We only keep the local maximum points with \(F_N(n) > \gamma_e\).

Denote the selected edges and the two ends of the whole frequency range by a set \(\Omega\), that is,
\[
\Omega = \{n | n\ is\ a\ local\ maximum\ point\ of\ F_N(n)\}
\]
and \(F_N(n) > \gamma_e \cup \{0, N-1\} \).

Let \(l_k\) (\(k = 1, 2, \cdots, K\)) be the elements of \(\Omega\) in ascending order. Thus \(l_k\) (\(k = 1, 2, \cdots, K\)) are the estimated band edges in the frequency range. The initially identified bands are therefore \([l_k, l_{k+1}]\), \(k = 1, 2, \cdots, K - 1\). Still some of the edges may be false band edges (caused by noise or signal spike). Our next step is to further classify the bands as occupied or vacant. This is a multi-band detection problem, though the bandwidths of the bands usually are different here.

Let \(\Delta(k)\) be the average PSD of the band \([l_k, l_{k+1}]\), that is,
\[
\Delta(k) = \frac{1}{l_{k+1} - l_k + 1} \sum_{n=l_k}^{l_{k+1}} P(n).
\]

We can compare \(\Delta(k)\) with the noise power to decide whether the band is occupied or vacant. However, the noise power is usually unknown and may change with time and location [21], [22], [23], [24]. So here we assume that the noise power is unknown and will estimate it using the wideband property.

The two hypotheses for each band are
\[
\begin{align*}
\mathcal{H}_0(k) &: \Delta(k) = \Delta_s(k) \\
\mathcal{H}_1(k) &: \Delta(k) = \Delta_s(k) + \Delta_d(k),
\end{align*}
\]

where \(\Delta_s(k)\) and \(\Delta_d(k)\) are the average PSD of source signal and noise within band \(k\), respectively. This is a multi-band sensing problem. We will discuss this in the next section.

If some of the detected occupied bands are consecutive, we combine them into one band. The noise power is then estimated as the average PSD of all detected vacant bands. Based on the estimated noise power, the average SNR of each occupied band can be easily estimated.

The method is summarized as follows.

**Algorithm 1: Edge based wideband sensing (EWS)**

1. Estimate the PSD of the received signal by (4).
2. Compute the product of the wavelet transforms at the first \(J\) dyadic scales as shown by (5).
3. Normalize the wavelet transform product by the mean of the PSD as shown in (7).
4. Find all the local maximum points of the function \(F_N(n)\) and denote them by set \(\Theta\).
5. For each local maximum point \(n\), compare \(F_N(n)\) with a threshold \(\gamma_e\) to obtain \(\Omega\), the set of indices of estimated edges and the two ends of the whole frequency range.
6. If there is at least one edge detected within the frequency range (excluding the two ends), continue the following steps; otherwise, the decision is \(\mathcal{H}_0\): “the whole frequency range is vacant”.
7. Form bands with the estimated edges as band ends and compute the average PSD of the bands.
8) Use a multi-band detection to classify each band as occupied or vacant. A multi-band detection will be discussed in detail in the next section.
9) If there is at least one band is detected as occupied, continue the following steps; otherwise, the decision is $\mathcal{H}_0$: “the whole frequency range is vacant”.
10) Combine consecutive occupied bands, if any, into one band.
11) Estimate the noise power as the average PSD of all detected vacant bands, and calculate the average SNR of each occupied band based on the estimated noise power.

IV. ENERGY COMPARISON MULTI-BAND SENSING

Based on the initial edge detection (Step 1 to 7 of Algorithm 1), we have formed the possible bands within the frequency range and obtained the average PSD of each band. Now we consider further classifying each band as occupied or vacant. This could be done by comparing the average PSD with the noise power, if the noise power is known a priori. Accurate knowledge on the noise power is thus the key of this approach. Unfortunately, in practice, noise uncertainty always exists, that is, noise power usually changes with time and location. Due to the noise uncertainty [21], [22], [23], [24], it is virtually impossible to obtain the accurate noise power. Inaccurate estimation of the noise power will cause this approach unreliable [21], [22], [23], [24]. To avoid this problem, we can use the special properties of large bandwidth for real-time noise power estimation. Within the large frequency range, it is very likely that there are a few vacant frequency intervals. So the average PSD on such intervals should approach to the noise power. The first metric is the $P_d$ and $P_{fa}$ for each band to evaluate the performance of a detection method. This is not enough for wideband sensing, as the band locations within the frequency range are unknown and the estimation accuracy of them is itself a critical performance metric. So except for the $P_d$ and $P_{fa}$ for the whole frequency range, we propose three criteria to evaluate the wideband sensing performance in the following. The three criteria evaluate a given method from different aspects.

A. Performance metrics

The first metric is the $P_d$ and $P_{fa}$ for the whole frequency range defined as

$$P_d = \mathbb{P} (\text{at least one band detected as occupied} | \mathcal{H}_1), \quad (13)$$

$$P_{fa} = \mathbb{P} (\text{at least one band detected as occupied} | \mathcal{H}_0), \quad (14)$$

where $\mathbb{P}(\xi)$ means the probability of a random event $\xi$.

We define the subcarrier occupancy index (SOI) of a subcarrier as 1 or 0, if the subcarrier is occupied or vacant. A natural way to evaluate the detection performance at subcarrier granularity is therefore the subcarrier occupancy error rate (SOER) defined as

$$\text{SOER} = \frac{\sum_{n=0}^{N-1} |\text{SOI}_a(n) - \text{SOI}_d(n)|}{N}, \quad (15)$$

where $\text{SOI}_a(n)$ and $\text{SOI}_d(n)$ are the actual and detected SOI for subcarrier $n$, respectively.

However, SOER does not take the signal strength into consideration. If the PSD of an occupied subcarrier is very small (near noise power), it is very likely that the subcarrier is detected as vacant and this is acceptable in many situations. At this case the detection error should be treated as smaller than that when an occupied subcarrier with large PSD being detected as vacant. To reflect this point, we give each subcarrier a subcarrier occupancy degree (SOD) defined as

$$\text{SOD}(n) = \left\{ \begin{array}{ll} v(n), & \text{subcarrier } n \text{ occupied} \\ 0, & \text{otherwise} \end{array} \right., \quad (16)$$

where $v(n)$ is the SNR of subcarrier $n$. The wideband spectral error (WSE) is defined as the mean square error of the SOD function, that is,

$$\text{WSE} = \frac{\sum_{n=0}^{N-1} |\text{SOD}_a(n) - \text{SOD}_d(n)|^2}{\sum_{n=0}^{N-1} |\text{SOD}_a(n)|^2}, \quad (17)$$

Algorithm 2: Energy comparison multi-band sensing (ECMS)

1) Reorder sequence $\Delta(k)$ into $\hat{\Delta}(k)$ with descending order.
2) Estimate the noise power by averaging on the PSDs of $K_s$ smallest bands:

$$\hat{\sigma}_n^2 = \frac{1}{K_s} \sum_{n=1}^{K_s} \hat{\Delta}(K - n). \quad (12)$$

3) For $p = K_s + 1, \cdots , K - 1$, if

$$\hat{\Delta}(K - p)/\hat{\sigma}_n^2 > \gamma_c,$$

then $K_v = p - 1$ and stop testing; otherwise, $K_v = p$ and continue testing for $p \leftarrow p + 1$; where $\gamma_c > 1$ is a threshold.
4) If $K_v < K - 1$, continue the following steps; otherwise, the decision is $\mathcal{H}_0$: “the whole frequency range is vacant”.
5) Find the $K_v$ bands with lowest powers and define them as vacant bands.
6) Combine consecutive occupied bands, if any, into a band.

IV. PERFORMANCE METRICS AND THRESHOLD

Performance evaluation in wideband sensing is much more complicated than that in single-band or multi-band sensing, where the band locations are known a priori. For single-band or multi-band sensing, we can use the $P_d$ and $P_{fa}$ for each band to evaluate the performance of a detection method. This is not enough for wideband sensing, as the band locations within the frequency range are unknown and the estimation accuracy of them is itself a critical performance metric. So except for the $P_d$ and $P_{fa}$ for the whole frequency range, we propose three criteria to evaluate the wideband sensing performance in the following. The three criteria evaluate a given method from different aspects.

A. Performance metrics

The first metric is the $P_d$ and $P_{fa}$ for the whole frequency range defined as

$$P_d = \mathbb{P} (\text{at least one band detected as occupied} | \mathcal{H}_1), \quad (13)$$

$$P_{fa} = \mathbb{P} (\text{at least one band detected as occupied} | \mathcal{H}_0), \quad (14)$$

where $\mathbb{P}(\xi)$ means the probability of a random event $\xi$.

We define the subcarrier occupancy index (SOI) of a subcarrier as 1 or 0, if the subcarrier is occupied or vacant. A natural way to evaluate the detection performance at subcarrier granularity is therefore the subcarrier occupancy error rate (SOER) defined as

$$\text{SOER} = \frac{\sum_{n=0}^{N-1} |\text{SOI}_a(n) - \text{SOI}_d(n)|}{N}, \quad (15)$$

where $\text{SOI}_a(n)$ and $\text{SOI}_d(n)$ are the actual and detected SOI for subcarrier $n$, respectively.

However, SOER does not take the signal strength into consideration. If the PSD of an occupied subcarrier is very small (near noise power), it is very likely that the subcarrier is detected as vacant and this is acceptable in many situations. At this case the detection error should be treated as smaller than that when an occupied subcarrier with large PSD being detected as vacant. To reflect this point, we give each subcarrier a subcarrier occupancy degree (SOD) defined as

$$\text{SOD}(n) = \left\{ \begin{array}{ll} v(n), & \text{subcarrier } n \text{ occupied} \\ 0, & \text{otherwise} \end{array} \right., \quad (16)$$

where $v(n)$ is the SNR of subcarrier $n$. The wideband spectral error (WSE) is defined as the mean square error of the SOD function, that is,

$$\text{WSE} = \frac{\sum_{n=0}^{N-1} |\text{SOD}_a(n) - \text{SOD}_d(n)|^2}{\sum_{n=0}^{N-1} |\text{SOD}_a(n)|^2}, \quad (17)$$
where SOD$_a(n)$ and SOD$_d(n)$ are the actual and detected SOD for subcarrier $n$, respectively.

These two criteria are at the subcarrier granularity. In practice, a source signal usually occupies a continuous frequency range (a band). Even if the average PSD of the band is high, at some subcarrier within the band the PSD may be low such that it is likely being detected as vacant. Hence we may be more interested in the detection accuracy on band granularity rather than on subcarrier granularity. For this, we define the band occupancy degree (BOD) as

$$\text{BOD}(k) = \begin{cases} \lambda(k), & \text{band } k \text{ occupied} \\ 0, & \text{otherwise} \end{cases},$$

where $\lambda(k)$ is the average SNR of band $k$. Each subcarrier $n$ should belong to one band, say, band $b(n)$, $1 \leq b(n) \leq L$, where $L$ is the total number of bands. The band occupancy error (BOE) is defined as

$$\text{BOE} = \frac{\sum_{n=0}^{N-1} |\text{BOD}_a(b_a(n)) - \text{BOD}_d(b_d(n))|^2}{\sum_{n=0}^{N-1} |\text{BOD}_a(b_a(n))|^2},$$

where BOA$_d(k)$ and BOA$_d(k)$ are the actual and detected BOD for band $k$, respectively.

In summary, $P_d$ and $P_{fa}$ only represent the detection performance in identifying the whole frequency range as occupied or vacant, while SOER, WSE and BOE give detailed detection performance within the frequency range.

### B. Threshold setting

As shown above, the performance metrics for wideband sensing is much more complicated. Theoretic analysis on the performance is therefore much more challenging.

Setting the thresholds to meet the pre-determined requirements is more difficult too. The threshold should be set such that certain requirements can be met. Since we have little or no information on the signal (actually we even do not know if there is a signal or not), usually we choose the threshold based on the $P_{fa}$ under hypothesis $\mathcal{H}_0$ (no signal case). The edge based wideband sensing has two detection phases: edge detection phase and multi-band detection phase. Hence it has two thresholds: one for the edge detection and one for the multi-band detection. Obviously, false alarm occurs if and only if at least one edge is detected at the edge detection phase and at least one band is classified as occupied at the multi-band detection phase. Thus we have

$$P_{fa} = P(\text{at least one band detected as occupied}\mid \mathcal{H}_0) = P\left(\max_n (f_N(n)) > \gamma_c \right) \text{ and } (\max_k (\Delta(k)) > \gamma_c \Lambda(K_s))\mid \mathcal{H}_0),$$

where $\Lambda(K_s)$ is the average value of least $K_s$ elements in $\Delta(k)$. If we can obtain a closed form expression of (20) as a function of $\gamma_c$ and $\gamma_e$, we can theoretically obtain the thresholds to meet the $P_{fa}$ requirement.

The distribution of $f(n)$ is discussed in [20], where a closed form expression is found for the special case of $J = 2$. For $J > 2$, it is an intractable mathematical problem to find a closed form solution. Simulation in [20] shows that multiple-term Gaussian mixture is a good approximation to the probability density function (PDF) of $f(n)$ in general. However, the parameters in the mixture needs to be determined by simulations. If the distribution of $f(n)$ is found, it is not difficult to find the distribution of $f_N(n)$. It is shown in [20] that $f(n)$ are essentially independent at different $n$ for white Gaussian input. Hence, the PDF of $\max_n (f_N(n))$ can be found given the PDF of $f_N(n)$. The most difficult part lies on the distribution of $\Delta(k)$ and $\max_k (\Delta(k))$. Note that the detected bands are related to the edge detection phase. Hence the distribution of $\Delta(k)$ must be conditioned on the possible edge locations detected at the first phase. This seems to be an intractable mathematical problem.

### VI. SIMULATIONS

We assume that the received wideband signal has been down-converted to the baseband. Two scenarios are considered here. **Scenario one**: the sensed bandwidth is $B_w = 100$ MHz and there are four occupied channels with bandwidth 6 MHz whose center frequencies (at baseband) are -37, -17, 13, 38 MHz, respectively. The signal at each occupied channel is a linear Chirp. **Scenario two**: the sensed bandwidth is 21.52 MHz and there are two occupied ATSC DTV channels with center frequencies -5.38 and 5.38 MHz, respectively. The bandwidth of each channel is supposed to be 6 MHz. However the edges of the channels are not sharp and therefore the actual bandwidth depends on interpretation. *Assume that the receiver (sensor) does not have any information on signal property, center frequencies, bandwidths and noise power.*

We define the wideband SNR of the received signal as the ratio of signal power to the noise power in the whole sensed frequency range. The SNR within each occupied channel is called channel SNR. The channel SNRs at different channels are usually different. In the simulations, at a fixed wideband SNR, the channel SNRs at scenario one are independently generated random numbers and different at different Monte-Carlo tests. For scenario two, the signal at each occupied channel is the captured DTV signal [25].

The FFT size is chosen as $N = 2048$ and sensing time (accumulated sampling time) is 5 ms (milli second). For ECMS, we choose $K_s = 1$. The threshold is set such that $P_{fa} = 0.0004$. The notations in the figures are defined in Section V. For convenience, we quote the definitions here: BOD: band occupancy degree; BOE: band occupancy error; SOER: subcarrier occupancy error rate; and WSE: wideband spectral error.

With wideband SNR at -10dB at scenario one, the received signal PSD, the actual BOD and the detected BOD ($J = 4$) are shown in Figure 1 (from left to right) at one Monte-Carlo test. At this test, the method well detected the four occupied bands (in terms of band edges and band SNR). The $P_d$ for the detection in the whole frequency range and the detailed performance for the proposed method are given in Figure 2 to Figure 3 (based on 200 Monte-Carlo tests). From the figures,
we see that larger $J$ is likely to give better performance, especially at low SNR, and the method usually gives good band location and BOD estimations for $J \geq 3$.

With wideband SNR at 0dB at scenario two, the received signal PSD, the actual BOD and the detected BOD are shown in Figure 4 (from left to right) at one Monte-Carlo test. The method detected two bands with bandwidth slightly smaller than 6MHz. This is due to the relatively smooth edges of the bands. Please note that method suppressed the single carrier noise at frequency point 0. This is a useful property as in practice there usually exist unwanted spurious signals generated by other electronic devices or other transmitters [26], [7]. It is desirable if they can be suppressed at the detection so that they do not cause false alarm. The $P_d$ for the detection in the whole frequency range and the detailed performance for the proposed method are given in Figure 5 to Figure 6 (based on 200 Monte-Carlo tests). The observations for scenario one are still valid here. However, the BOD estimation is in general worse than that for scenario one as the band edges are not sharp.

VII. CONCLUSIONS

In this paper, a practical method has been proposed for sensing a very large frequency range. The method includes
two major steps: edge detection using multi-scale wavelet transform and multi-band detection using energy comparison. New performance metrics have also been discussed. Simulations have shown that the method is effective at certain situations. Further research is needed to conduct theoretical analysis on the methods as there are some mathematically intractable problems with them.

REFERENCES


